# Network Compression

AI602: Recent Advances in Deep Learning

Lecture 9

Slide made by

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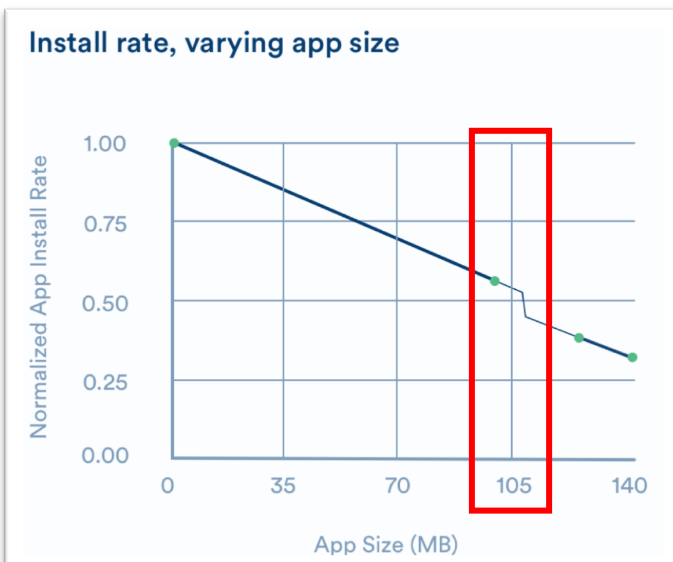
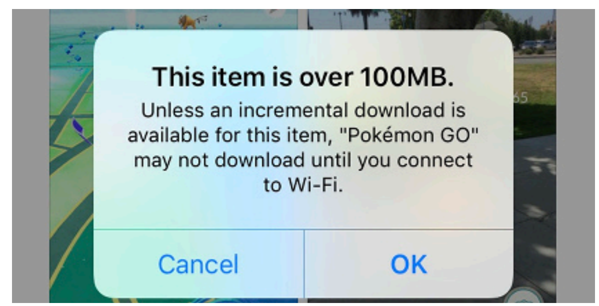
KAIST EE

- **Deploying deep neural networks** (DNNs) has been increasingly difficult
  - Constraints on power consumption, memory usage, inference overhead, …
- Inference with a large-scale network consumes huge costs
- In mobile apps, such issues become more serious
  - **“The dreaded 100MB effect”**
- Can we make DNNs to perform inferences more efficiently?





*source: https://www.recode.net/2016/10/4/13151432/app-size-calculator-bloat-experiment-developers-segment

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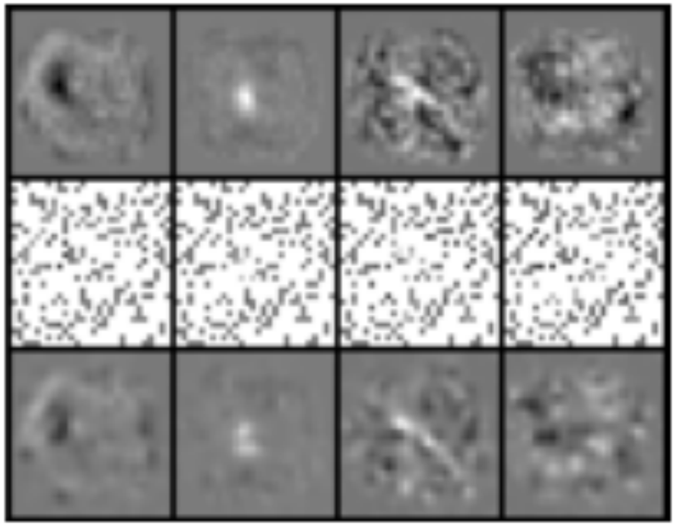
# Redundancies in Deep Neural Networks [Denil et al., 2013]

- DNNs include a significant number of **redundant parameters**
- Denil et al. (2013): Predicting > 95% of weights from < 5%
  - A simple kernel ridge regression is sufficient
  - … without any drop in accuracy!
  - Many of the weights **need not be learned at all**

(a) Original weights

(b) Randomly selected

(c) Predicted from (b)



- Such redundancy can be exploited via **network pruning**

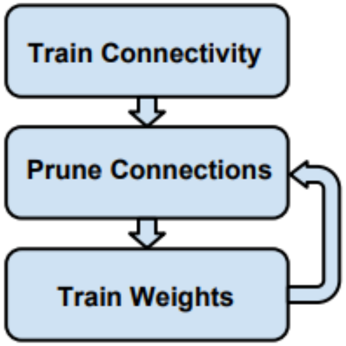


*source: Denil et al., "Predicting Parameters in Deep Learning", NIPS 2013

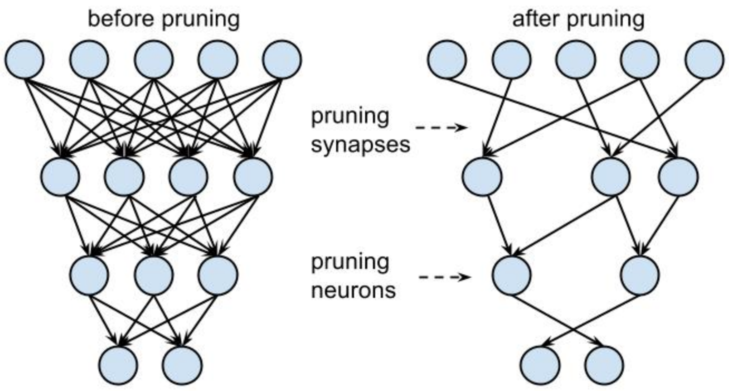
# Network Pruning

- Determining **low-saliency parameters**, given a pre-trained network
- Follows the framework proposed by LeCun et al. (1990):

1. **Train** a deep model until convergence
2. **Delete** “unimportant” connections w.r.t. a certain criteria
3. **Re-train** the network
4. **Iterate** to step 2, or stop




- Defining which connection is unimportant can vary
    - Weight magnitudes ($L^2$, $L^1$, …)
    - Mean activation [Molchanov et al., 2016]
    - Avg. % of Zeros (APoZ) [Hu et al., 2016]
    - Low entropy activation [Luo et al., 2017]
    - …






*source: LeCun et al., “Optimal Brain Damage”, NIPS 1990

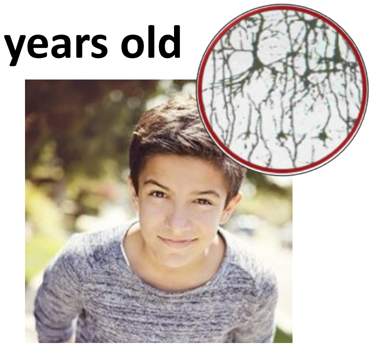
# Synaptic Pruning in Human Brain

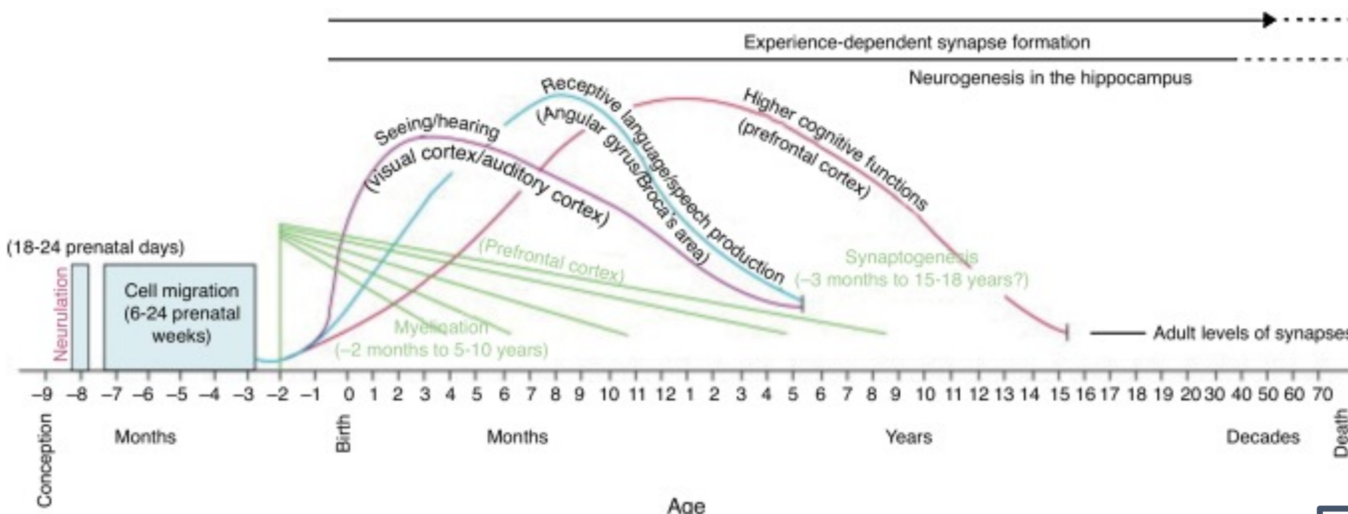
- Human brains are also using pruning schemes as well
- **Synaptic pruning** removes redundant synapses in the brain during lifetime

**At birth**

**6 years old**

**14 years old**






Next: OBD

*source: Leisman et al., “The neurological development of the child with the educational enrichment in mind.”, Psicología Educativa 2015

## Optimal Brain Damage (OBD) [LeCun et al., 1990]

- Network pruning **perturbs weights $\mathbf{W}$** by **zeroing** some of them
- How the loss $L$ would be changed when $\mathbf{W}$ is perturbed?

- **OBD** approximates $L$ by the 2nd order Taylor series:

$$\delta L \simeq \underbrace{\sum_i \frac{\partial L}{\partial w_i}\delta w_i}_{\text{1st order}} + \underbrace{\frac{1}{2}\sum_i \frac{\partial^2 L}{\partial {w_i}^2}\delta w_i^2 + \frac{1}{2}\sum_{i,j} \frac{\partial^2 L}{\partial w_i \partial w_j}\delta w_i \delta w_j}_{\text{2nd order}} + O(||\delta \mathbf{W}||^3)$$

- **Problem:** Computing $H = \left(\frac{\partial L}{\partial w_i \partial w_j}\right)_{i,j}$ is usually intractable
  - Requires $O(n^2)$ on # weights
  - Neural networks usually have enormous number of weights
    - e.g. AlexNet: **60M** parameters $\Rightarrow$ $H$ consists $\approx \mathbf{3.6 \times 10^{15}}$ elements



*source: LeCun et al., "Optimal Brain Damage", NIPS 1990

# Optimal Brain Damage (OBD) [LeCun et al., 1990]

- **Problem:** Computing $H = \left(\frac{\partial L}{\partial w_i \partial w_j}\right)_{i,j}$ is usually intractable

- Two additional assumptions for tractability

  1. **Diagonal** approximation: $H = \frac{\partial^2 L}{\partial w_i \partial w_j} = 0 \quad \text{if} \quad i \neq j$
  2. **Extremal** assumption: $\frac{\partial L}{\partial w_i} = 0 \quad \forall i$
     - **W** would be in a local minima if it's pre-trained

- Now we get: $\delta L \simeq \frac{1}{2} \sum_i \frac{\partial^2 L}{\partial {w_i}^2} \delta w_i^2 + O(||\delta \mathbf{W}||^3)$
  - It only needs $\mathrm{diag}(H) := \left(\frac{\partial^2 L}{\partial w_i^2}\right)_i$

- $\mathbf{diag}(\boldsymbol{H})$ can be computed in $O(n)$, allowing a backprop-like algorithm
  - For details, see [LeCun et al., 1987]



*source: LeCun et al., "Optimal Brain Damage", NIPS 1990

# Optimal Brain Damage (OBD) [LeCun et al., 1990]

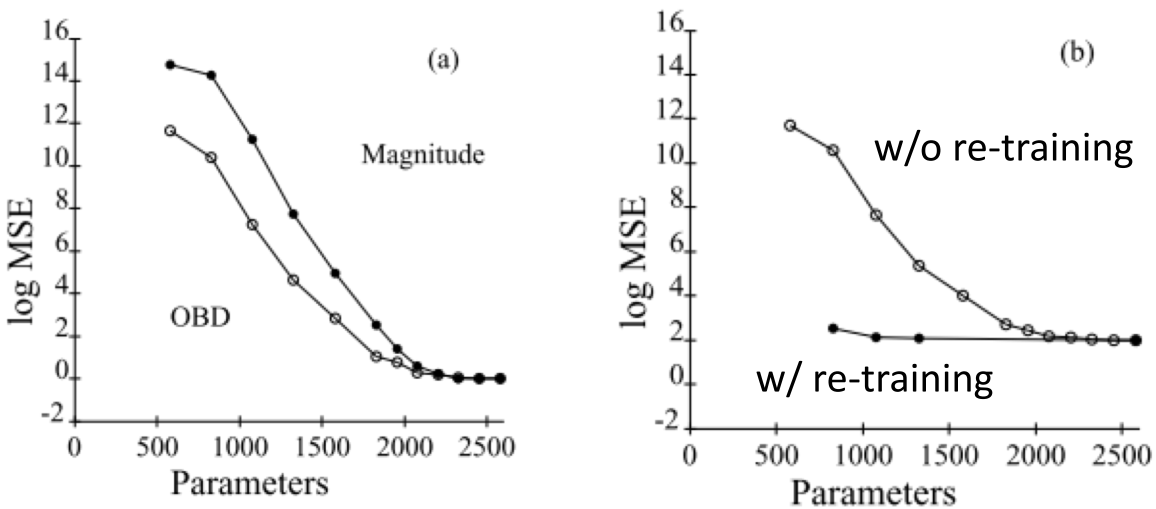
- How the loss $L$ would be changed when $\mathbf{W}$ is perturbed?

$$L(\delta\mathbf{W}) \simeq \frac{1}{2}\sum_i \frac{\partial^2 L}{\partial {w_i}^2}\delta w_i^2 =: \sum_i \frac{1}{2}h_{ii}\delta w_i^2$$

- The **saliency** for each weight ⇒ $s_i := \frac{1}{2}h_{ii}|w_i|^2$

$\boxed{s_i := |w_i|}$

- OBD shows robustness on pruning compared to magnitude-based deletion
- After re-training, the original test accuracy is recovered



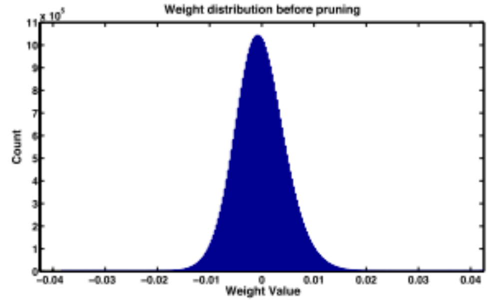

Next: Pruning modern DNNs

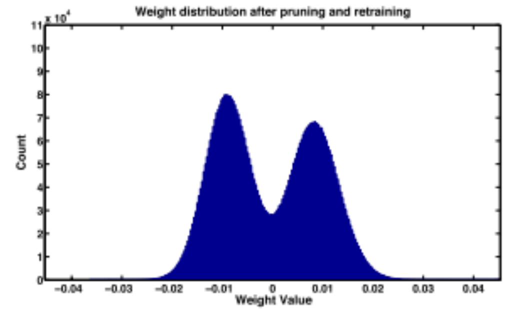


*source: LeCun et al., “Optimal Brain Damage”, NIPS 1990

## Pruning Modern DNNs [Han et al., 2015]

- Han et al. (2015): Pruning larger DNNs
  - LeNet, AlexNet, VGG-16, … on ImageNet
  - Highlights the **practical efficiency of pruning**
- OBD introduces extra computation on larger models
  - It requires an additional, separated backward pass
- The simple **magnitude-based pruning** works very well as long as the network is re-trained




Comparison with other model reduction methods on AlexNet

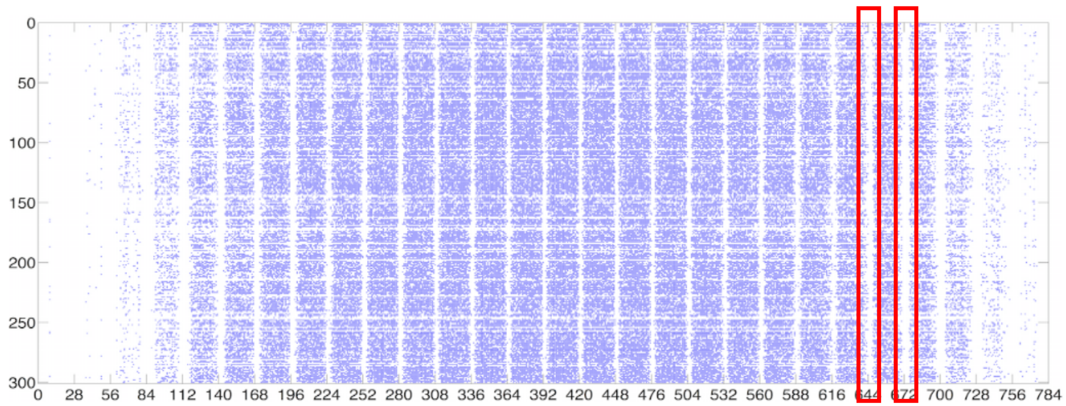
| Network | Top-1 Error | Top-5 Error | Parameters | Compression Rate |
|---|---|---|---|---|
| Baseline Caffemodel [26] | 42.78% | 19.73% | 61.0M | 1× |
| Data-free pruning [28] | 44.40% | - | 39.6M | 1.5× |
| Fastfood-32-AD [29] | 41.93% | - | 32.8M | 2× |
| Fastfood-16-AD [29] | 42.90% | - | 16.4M | 3.7× |
| Collins & Kohli [30] | 44.40% | - | 15.2M | 4× |
| Naive Cut | 47.18% | 23.23% | 13.8M | 4.4× |
| SVD [12] | 44.02% | 20.56% | 11.9M | 5× |
| **Network Pruning** | **42.77%** | **19.67%** | **6.7M** | **9×** |

*source: Han et al., “Learning both Weights and Connections for Efficient Neural Networks”, NIPS 2015

- Han et al. (2015): Pruning larger DNNs
  - Highlights the **practical efficiency of pruning**
- The **magnitude-based pruning** works well as long as the network is re-trained

| Network | Top-1 Error | Top-5 Error | Parameters | Compression Rate |
|---|---|---|---|---|
| LeNet-300-100 Ref | 1.64% | - | 267K | |
| LeNet-300-100 Pruned | 1.59% | - | **22K** | **12**× |
| LeNet-5 Ref | 0.80% | - | 431K | |
| LeNet-5 Pruned | 0.77% | - | **36K** | **12**× |
| AlexNet Ref | 42.78% | 19.73% | 61M | |
| AlexNet Pruned | 42.77% | 19.67% | **6.7M** | **9**× |
| VGG-16 Ref | 31.50% | 11.32% | 138M | |
| VGG-16 Pruned | 31.34% | 10.88% | **10.3M** | **13**× |

- Network pruning detects visual attention regions






*source: Han et al., "Learning both Weights and Connections for Efficient Neural Networks", NIPS 2015

# Pruning Modern DNNs

- The **magnitude-based pruning** works well as long as the network is re-trained

- Mittal et al. (2018): In fact, **pruning criteria are not that important**
  - … as long as the re-training phase exists
- Many strategies cannot even beat **random pruning** after fine-tuning

| Heuristic | 25 % | 50% | 75% |
|---|---|---|---|
| Random | 0.650 | 0.569 | 0.415 |
| Mean Activation | 0.652 | 0.570 | 0.409 |
| Entropy | 0.641 | 0.549 | 0.405 |
| Scaled Entropy | 0.637 | 0.550 | 0.401 |
| $l_1$-norm | **0.667** | **0.593** | **0.436** |
| APoZ | 0.647 | 0.564 | 0.422 |
| Sensitivity | 0.636 | 0.543 | 0.379 |

Table 1: Comparison of different filter pruning strategies on VGG-16.

| Heuristics | #Layers Pruned | 25 % | 50% | 75% |
|---|---|---|---|---|
| Random | 16 | 0.722 | 0.683 | 0.617 |
| $l_1$-norm | 16 | 0.714 | 0.677 | 0.610 |
| Random | 32 | 0.696 | 0.637 | 0.518 |
| $l_1$-norm | 32 | 0.691 | 0.633 | 0.514 |

Table 3: Comparison of different filter pruning strategies on ResNet (Top-1 accuracy of unpruned network is 0.745)

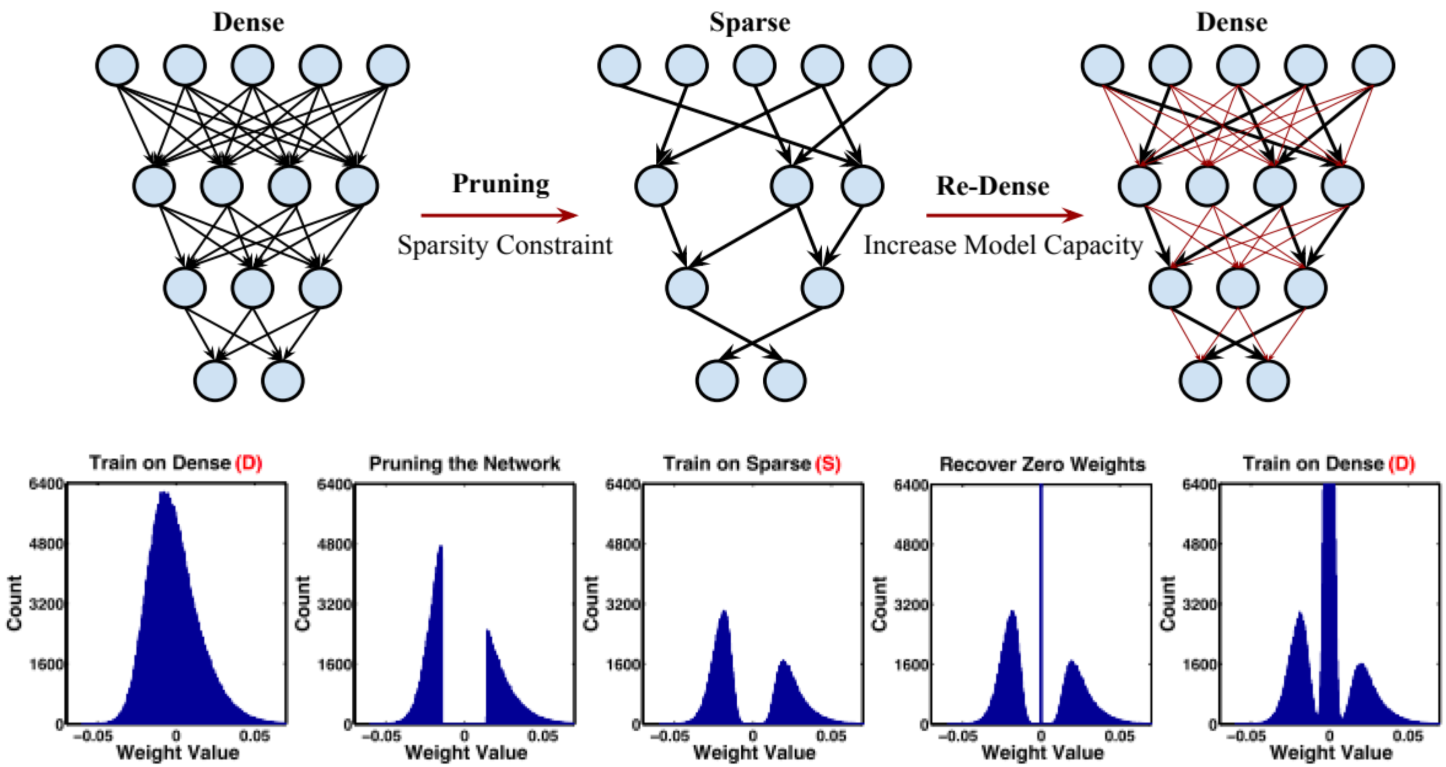
- The compressibility of DNNs are NOT due to the specific criterion
  - … but due to the inherent plasticity of DNNs

Next: Dense-Sparse-Dense

*source: Mittal et al., "Recovering from Random Pruning: On the Plasticity of Deep Convolutional Neural Networks", WACV 2018

# Network Re-wiring: Dense-Sparse-Dense Training Flow

- Network pruning preserves accuracy of the original network
- Han et al. (2017): Re-wiring the pruned connections improves DNNs further
  - **"Dense-Sparse-Dense"** training flow




*source: Han et al., "DSD: Dense-Sparse-Dense Training for Deep Neural Networks", ICLR 2017

# Network Re-wiring: Dense-Sparse-Dense Training Flow

- Network pruning preserves accuracy of the original network
- Han et al. (2017): Re-wiring the pruned connections improves DNNs further
  - **"Dense-Sparse-Dense"** training flow

- Pruning discovers better optimum that the current training cannot find

| Neural Network | Domain | Dataset | Type | Baseline | DSD | Abs. Imp. | Rel. Imp. |
|---|---|---|---|---|---|---|---|
| GoogLeNet | Vision | ImageNet | CNN | 31.1%[1] | **30.0%** | 1.1% | 3.6% |
| VGG-16 | Vision | ImageNet | CNN | 31.5%[1] | **27.2%** | 4.3% | 13.7% |
| ResNet-18 | Vision | ImageNet | CNN | 30.4%[1] | **29.2%** | 1.2% | 4.1% |
| ResNet-50 | Vision | ImageNet | CNN | 24.0%[1] | **22.9%** | 1.1% | 4.6% |
| NeuralTalk | Caption | Flickr-8K | LSTM | 16.8[2] | **18.5** | 1.7 | 10.1% |
| DeepSpeech | Speech | WSJ'93 | RNN | 33.6%[3] | **31.6%** | 2.0% | 5.8% |
| DeepSpeech-2 | Speech | WSJ'93 | RNN | 14.5%[3] | **13.4%** | 1.1% | 7.4% |

*source: Han et al., "DSD: Dense-Sparse-Dense Training for Deep Neural Networks", ICLR 2017

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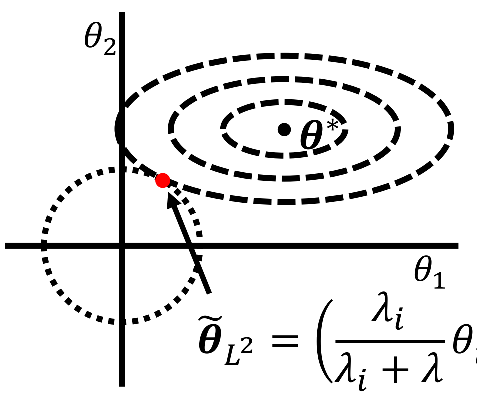
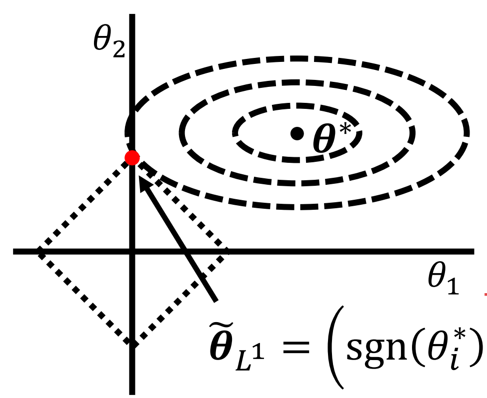
- The performance of pruning depends on the **initial training scheme**
  - e.g. Which regularization to use: $L^2$ or $L^1$?
- Which training scheme will maximize the pruning performance?
  - We still don't know about the optimal points of a DNN

- One prominent way: **Sparse network learning**
  - Inducing to a sparse solution from training a network
  - Weights with value 0 can safely be removed ⇒ it **does not** require re-training
- **Example**: $L^1$-regularization




$$\tilde{\boldsymbol{\theta}}_{L^2} = \left(\underline{\frac{\lambda_i}{\lambda_i + \lambda}}\theta_i^*\right)_i$$

"rescaling" eigenvalues



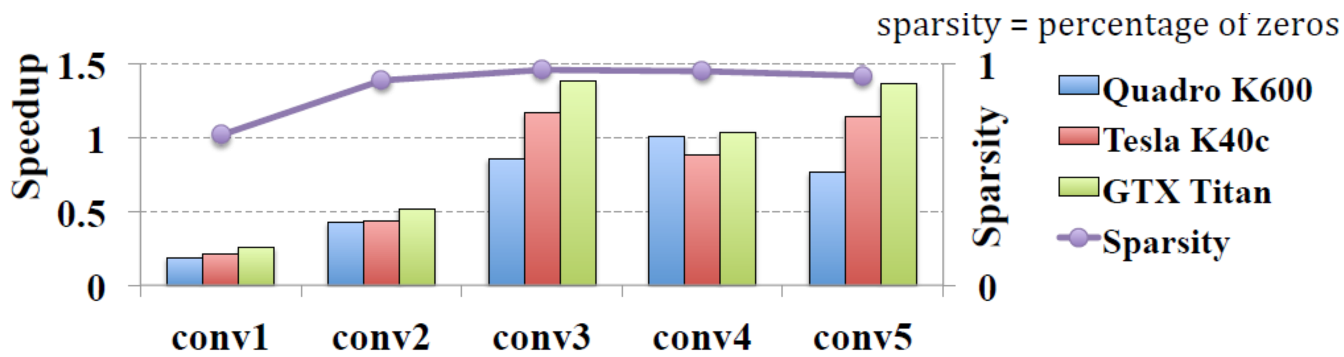

"**sparse**" solution

$$\tilde{\boldsymbol{\theta}}_{L^1} = \left(\operatorname{sgn}(\theta_i^*)\max\left\{|\theta_i^*| - \frac{\lambda}{\lambda_i}, 0\right\}\right)_i$$

Next: Structured sparsity learning

# Structured Sparsity Learning [Wen et al., 2016]

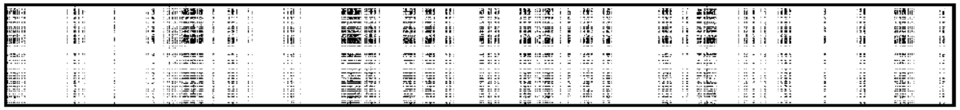
- "Un-structured" weight-level pruning may not engage a **practical speed-up**
  - Despite of extremely high sparsity, actual speed-ups in GPU is limited




**Speed-up ratio of weight-level pruning**

Non-structured sparsity (poor data pattern)

Structured sparsity (regular data pattern)



5× speedup after concatenation of nonzero rows and columns



*source: Wen et al., "Learning structured sparsity in deep neural networks." NIPS 2016

- **Structured sparsity** can be induced by adding group-lasso regularization

$$\min_{\mathbf{W}} \mathcal{L}(\mathbf{W}) + \lambda \sum_{l=1}^{L} R_g(\mathbf{W}^{(l)}), \;\; R_g(\mathbf{w}) = \sum_{g=1}^{G} \|\mathbf{w}^{(g)}\|_2$$

\# groups

  - **Filter-wise** and **channel-wise**: # filters # channels

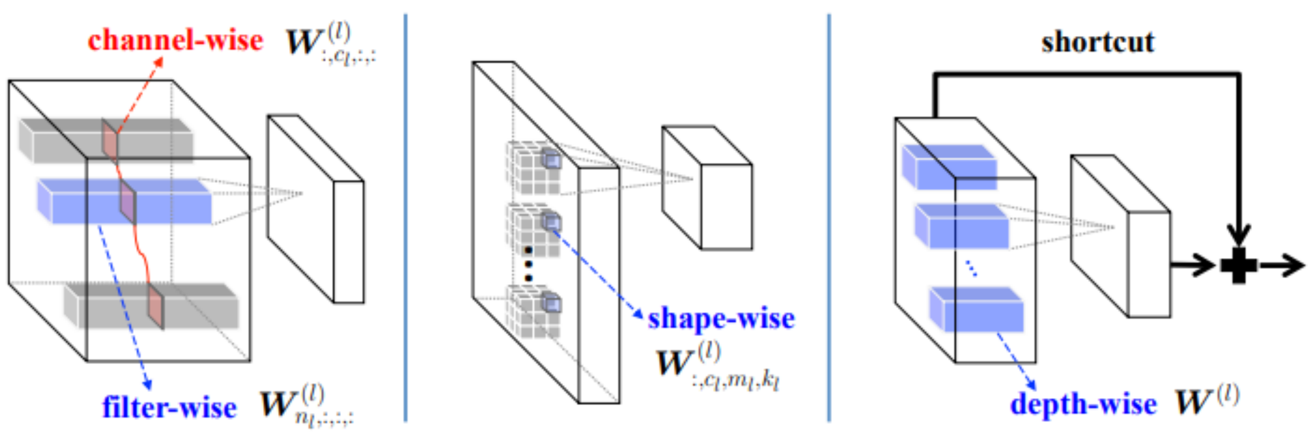
$$R_g(\mathbf{W}^{(l)}) = \sum_{n_l=1}^{N_l} \|\mathbf{W}^{(l)}_{n_l,:,:,:}\|_2 + \sum_{c_l=1}^{C_l} \|\mathbf{W}^{(l)}_{:,c_l,:,:}\|_2$$

  - **Shape-wise** sparsity: width height

$$R_g(\mathbf{W}^{(l)}) = \sum_{c_l=1}^{C_l} \sum_{m_l=1}^{M_l} \sum_{k_l=1}^{K_l} \|\mathbf{W}^{(l)}_{:,c_l,m_l,k_l}\|_2$$

  - **Depth-wise** sparsity (applicable only for ResNet):

$$R_g(\mathbf{W}^{(l)}) = \|\mathbf{W}^{(l)}\|_2$$





*source: Wen et al., “Learning structured sparsity in deep neural networks.” NIPS 2016

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  - **Filter-wise** and **channel-wise**:

# filters

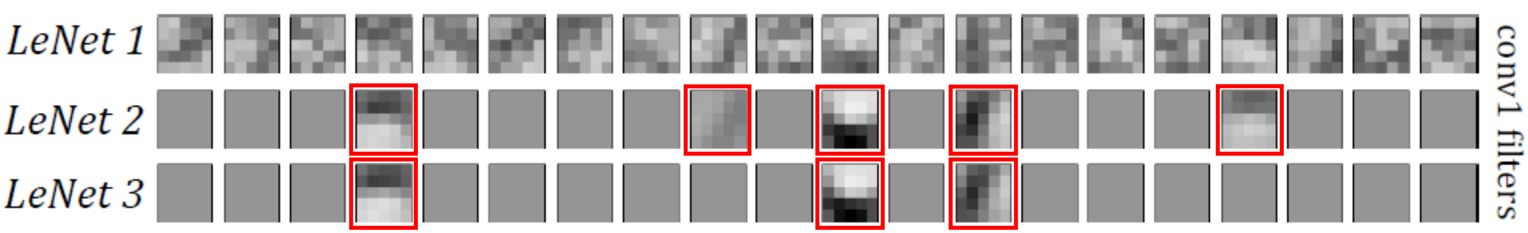
# channels

$$R_g(\mathbf{W}^{(l)}) = \sum_{n_l=1}^{N_l} \|\mathbf{W}^{(l)}_{n_l,:,:,:}\|_2 + \sum_{c_l=1}^{C_l} \|\mathbf{W}^{(l)}_{:,c_l,:,:}\|_2$$

Table 1: Results after penalizing unimportant filters and channels in *LeNet*

| *LeNet* # | Error | Filter # § | Channel # § | FLOP § | Speedup § |
|---|---|---|---|---|---|
| 1 (*baseline*) | 0.9% | 20—50 | 1—20 | 100%—100% | 1.00×—1.00× |
| 2 | 0.8% | 5—19 | 1—4 | 25%—7.6% | 1.64×—5.23× |
| 3 | 1.0% | 3—12 | 1—3 | 15%—3.6% | 1.99×—7.44× |

§In the order of *conv1*—*conv2*




Fewer but smoother feature extractors



*source: Wen et al., "Learning structured sparsity in deep neural networks." NIPS 2016

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$$\min_{\mathbf{W}} \mathcal{L}(\mathbf{W}) + \lambda \sum_{l=1}^{L} R_g(\mathbf{W}^{(l)}), \;\; R_g(\mathbf{w}) = \sum_{g=1}^{G} \|\mathbf{w}^{(g)}\|_2$$

  - **Shape-wise** sparsity:

width height

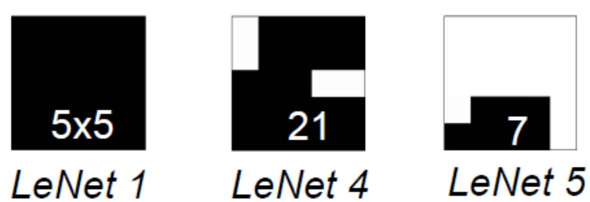
$$R_g(\mathbf{W}^{(l)}) = \sum_{c_l=1}^{C_l} \sum_{m_l=1}^{M_l} \sum_{k_l=1}^{K_l} \|\mathbf{W}^{(l)}_{:,c_l,m_l,k_l}\|_2$$

Table 2: Results after learning filter shapes in *LeNet*

| *LeNet* # | Error | Filter size § | Channel # | FLOP | Speedup |
|---|---|---|---|---|---|
| 1 (*baseline*) | 0.9% | 25—500 | 1—20 | 100%—100% | 1.00×—1.00× |
| 4 | 0.8% | 21—41 | 1—2 | 8.4%—8.2% | 2.33×—6.93× |
| 5 | 1.0% | 7—14 | 1—1 | 1.4%—2.8% | 5.19×—10.82× |

§ The sizes of filters after removing zero shape fibers, in the order of *conv1—conv2*

Learned shapes of conv1 filters:






*source: Wen et al., "Learning structured sparsity in deep neural networks." NIPS 2016

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- **Structured sparsity** can be induced by adding group-lasso regularization

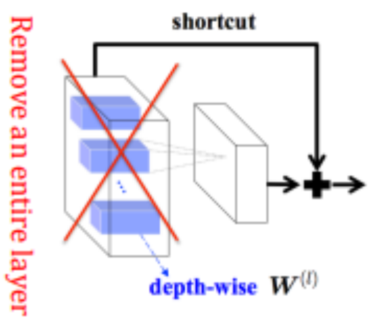
$$\min_{\mathbf{W}} \mathcal{L}(\mathbf{W}) + \lambda \sum_{l=1}^{L} R_g(\mathbf{W}^{(l)}), \;\; R_g(\mathbf{w}) = \sum_{g=1}^{G} \|\mathbf{w}^{(g)}\|_2$$

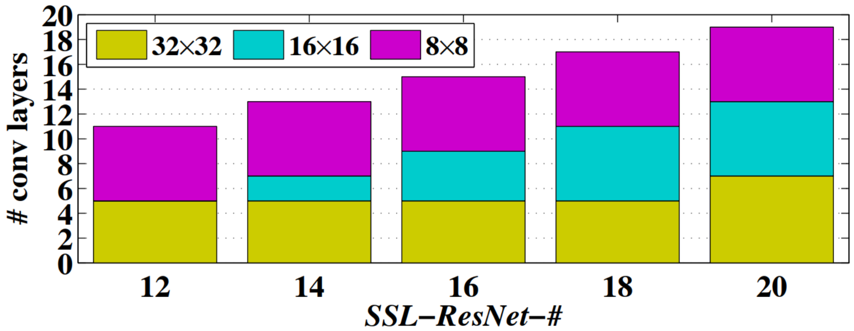
  - **Depth-wise** sparsity: $R_g(\mathbf{W}^{(l)}) = \|\mathbf{W}^{(l)}\|_2$

ResNet-20/32: baseline with 20/32 layers

SSL-ResNet-#: Ours with # layers after learning depth of ResNet-20

| | # layers | error | # layers | error |
|---|---|---|---|---|
| ResNet | 20 | 8.82% | 32 | 7.51% |
| SSL-ResNet | **14** | **8.54%** | **18** | **7.40%** |




Next: Sparsification via variational dropout



*source: Wen et al., "Learning structured sparsity in deep neural networks." NIPS 2016

# Recall: Variational Dropout [Kingma et al., 2015]

- **Variational dropout** (VD) allows to learn the dropout rates separately

- Unlike dropout, VD imposes noises on **weights** $\boldsymbol{\theta}$:

$$w_i := \theta_i \cdot \xi_i, \qquad \text{where} \quad p_{\alpha_i}(\xi_i) = \mathcal{N}(1, \alpha_i)$$

  - A Bayesian generalization of Gaussian dropout [Srivastava et al., 2014]
  - $\mathbf{w} = (w_i)_i$ is adapted to data in Bayesian sense by optimizing $\boldsymbol{\alpha}$ and $\boldsymbol{\theta}$

- **Re-parametrization trick** allows $\mathbf{w}$ to be learned via minibatch-based gradient estimation methods [Kingma & Welling, 2013]
  - $\boldsymbol{\alpha}$ and $\boldsymbol{\theta}$ can be optimized separated from noises

$$w_i = \theta_i + (\theta_i \sqrt{\alpha_i}) \cdot \varepsilon_i, \qquad \text{where} \quad \varepsilon_i \sim \mathcal{N}(0, 1)$$



*source : Kingma et al., "Variational dropout and the local reparametrization trick", NIPS 2015

## Variational Dropout Sparsifies DNNs [Molchanov et al., 2017]

- VD imposes noises on weights $\theta$:

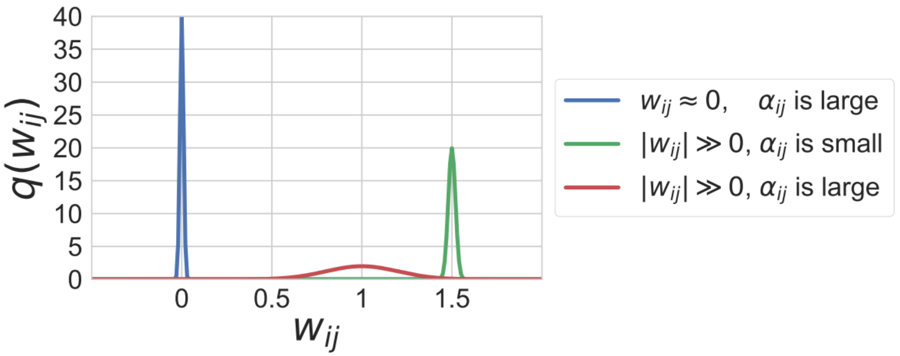
$$w_i := \theta_i \cdot \xi_i, \qquad \text{where} \quad p_{\alpha_i}(\xi_i) = \mathcal{N}(1, \boxed{\alpha_i})$$

- The original VD set a constraint $\alpha_i \leq 1$ for technical reasons
  - It corresponds to $p \leq 0.5$ in binary dropout

**Q. What if $\alpha_i > 1$? What happens when $\alpha_i \to \infty$?**

- $p(w_i) = \theta_i \cdot p(\xi_i) = \mathcal{N}(\theta_i, \alpha_i \theta_i^2)$
- $w_i$ will be completely random as $\alpha_i \to \infty$
- Such $w_i$ will **corrupt** the expected log likelihood
- … except that $\theta_i \to 0$ as well!

$$\boxed{\begin{gathered}\theta_{ij} \to 0, \quad \alpha_{ij}\theta_{ij}^2 \to 0 \\ \Downarrow \\ q(w_{ij} \,|\, \theta_{ij}, \alpha_{ij}) \to \mathcal{N}(w_{ij} \,|\, 0, 0) = \delta(w_{ij})\end{gathered}}$$



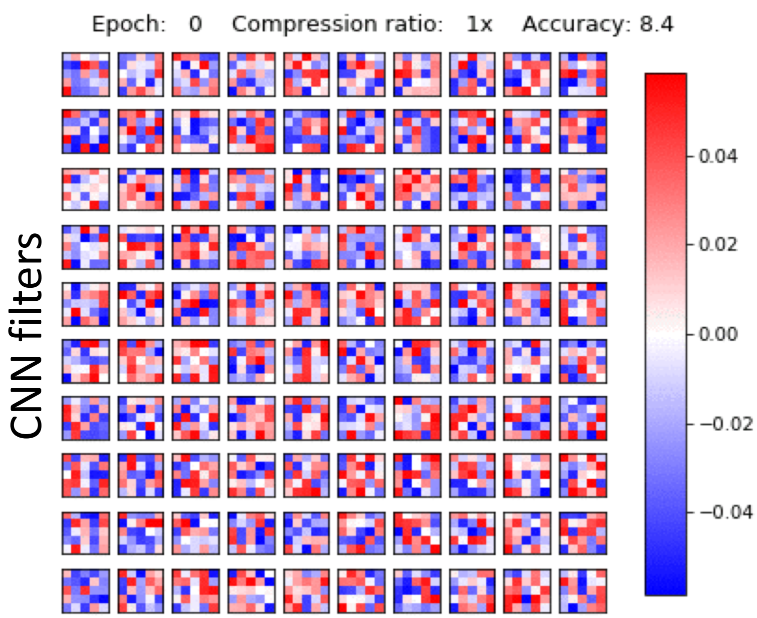


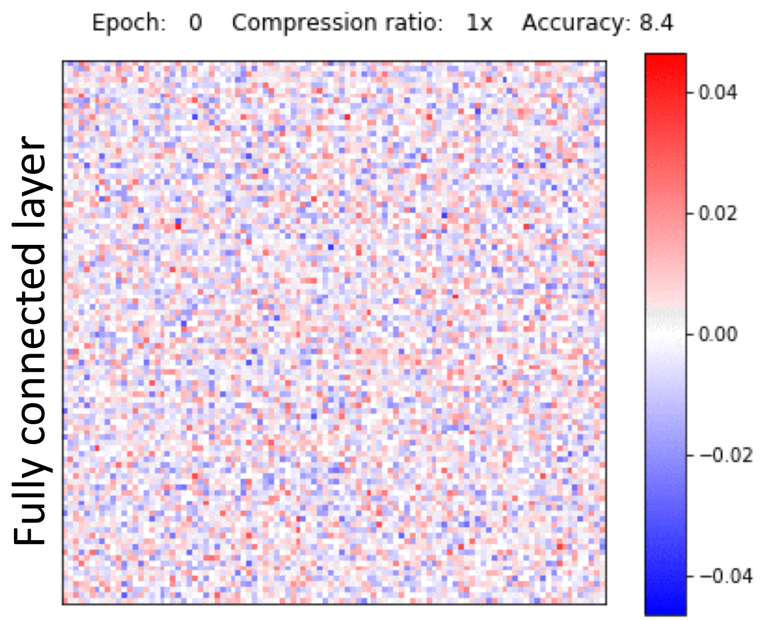

*source : Molchanov et al., "Variational Dropout Sparsifies Deep Neural Networks", ICML 2017

**Q.** What if $\alpha_i > 1$? What happens when $\alpha_i \to \infty$?

- It will **corrupt** the expected log likelihood except that $\theta_i \to 0$ as well

- Molchanov et al. (2017): **Extending VD for** $\alpha_i > 1$ ⇒ Super sparse solutions
  - Weights with $\log \alpha > 3$ are pruned away during training





*source : Molchanov et al., "Variational Dropout Sparsifies Deep Neural Networks", ICML 2017

**Q.** What if $\alpha_i > 1$? What happens when $\alpha_i \to \infty$?

- It will **corrupt** the expected log likelihood except that $\theta_i \to 0$ as well

- Molchanov et al. (2017): **Extending VD for** $\alpha_i > 1 \Rightarrow$ Super sparse solutions
  - Weights with $\log \alpha > 3$ are pruned away during training

| Network | Method | Error % | Sparsity per Layer % | $\frac{\lvert\mathbf{W}\rvert}{\lvert\mathbf{W}_{\neq 0}\rvert}$ | |
|---|---|---|---|---|---|
| | Original | 1.64 | | 1 | |
| | Pruning | 1.59 | $92.0 - 91.0 - 74.0$ | 12 | [Han et al., 2015] |
| LeNet-300-100 | DNS | 1.99 | $98.2 - 98.2 - 94.5$ | 56 | |
| | SWS | 1.94 | | 23 | |
| (ours) | Sparse VD | 1.92 | $98.9 - 97.2 - 62.0$ | **68** | |
| | Original | 0.80 | | 1 | |
| | Pruning | 0.77 | $34 - 88 - 92.0 - 81$ | 12 | [Han et al., 2015] |
| LeNet-5-Caffe | DNS | 0.91 | $86 - 97 - 99.3 - 96$ | 111 | |
| | SWS | 0.97 | | 200 | |
| (ours) | Sparse VD | 0.75 | $67 - 98 - 99.8 - 95$ | **280** | |

Next: Variational information bottleneck



*source : Molchanov et al., "Variational Dropout Sparsifies Deep Neural Networks", ICML 2017

## Variational Information Bottleneck [Dai et al., 2018]

- **Motivation**: Markov chain interpretation of DNN [Tishby & Zaslavsky, 2015]

$$\boldsymbol{y} \to \boldsymbol{x} = \boldsymbol{h}_0 \to \boldsymbol{h}_1 \to \cdots \to \underbrace{\boldsymbol{h}_{i-1} \to \boldsymbol{h}_i}_{p(\boldsymbol{h}_i|\boldsymbol{h}_{i-1})} \to \cdots \to \underline{\boldsymbol{h}_L \to \hat{\boldsymbol{y}}}$$

Approximate $p(\boldsymbol{y}|\boldsymbol{h}_L)$
via tractable $p(\hat{\boldsymbol{y}}|\boldsymbol{h}_L)$


1. **Maximize** $I(\boldsymbol{h}_i; \boldsymbol{y})$ for high-accuracy prediction
2. **Minimize** $I(\boldsymbol{h}_i; \boldsymbol{h}_{i-1})$ for compression ⇒ **"information bottleneck"**

- **Layer-wise losses** become:

Mutual information

$$\mathcal{L}_i = \gamma_i I(\boldsymbol{h}_i; \boldsymbol{h}_{i-1}) - I(\boldsymbol{h}_i; \boldsymbol{y})$$

The relative strength of bottleneck



*source : Dai et al., "Compressing Neural Networks using the Variational Information Bottleneck", ICML 2018

## Variational Information Bottleneck [Dai et al., 2018]

- **Layer-wise losses** become $\mathcal{L}_i = \gamma_i I(\boldsymbol{h}_i; \boldsymbol{h}_{i-1}) - I(\boldsymbol{h}_i; \boldsymbol{y})$
- **Problem**: Computing $I(\cdot;\cdot)$ is usually intractable
- Instead, we minimize variational upper bound of it

$$\mathcal{L}_i \leq \tilde{\mathcal{L}}_i = \gamma_i \mathbb{E}[\mathrm{KL}(\boxed{p(\boldsymbol{h}_i|\boldsymbol{h}_{i-1})}||\boxed{q(\boldsymbol{h}_i)})] - \mathbb{E}[\log \boxed{q(\boldsymbol{y}|\boldsymbol{h}_L)}]$$

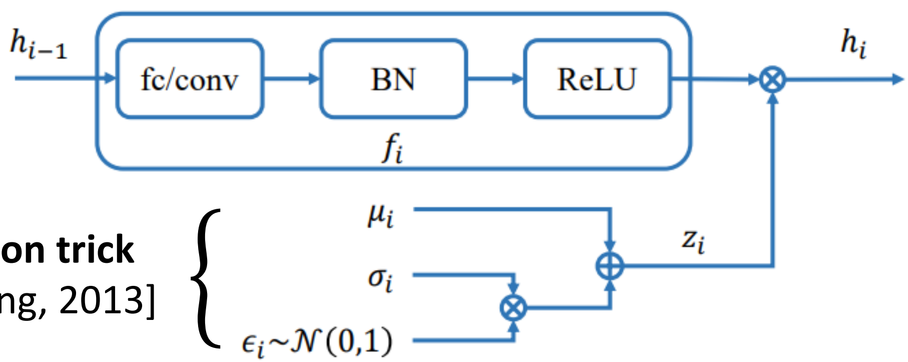
variational approx. of $p(\boldsymbol{h}_i)$

variational approx. of $p(\boldsymbol{y}|\boldsymbol{h}_L)$
$\begin{cases} \textbf{multinomal} \text{ for classification} \\ \textbf{Gaussian} \text{ for regression} \end{cases}$

- **Variational Information Bottleneck (VIB)** model

$$p(\boldsymbol{h}_i|\boldsymbol{h}_{i-1}) := f_i(\boldsymbol{h}_{i-1}) \odot \mathcal{N}(\boldsymbol{h}_i|\boldsymbol{\mu}_i, \mathrm{diag}(\boldsymbol{\sigma}_i^2))$$
$$q(\boldsymbol{h}_i) := \mathcal{N}(\boldsymbol{h}_i|\boldsymbol{0}, \mathrm{diag}(\boldsymbol{\xi}_i))$$

**Reparametrization trick**
[Kingma & Welling, 2013]






*source : Dai et al., "Compressing Neural Networks using the Variational Information Bottleneck", ICML 2018

- We minimize variational upper bound of $\mathcal{L}_i$

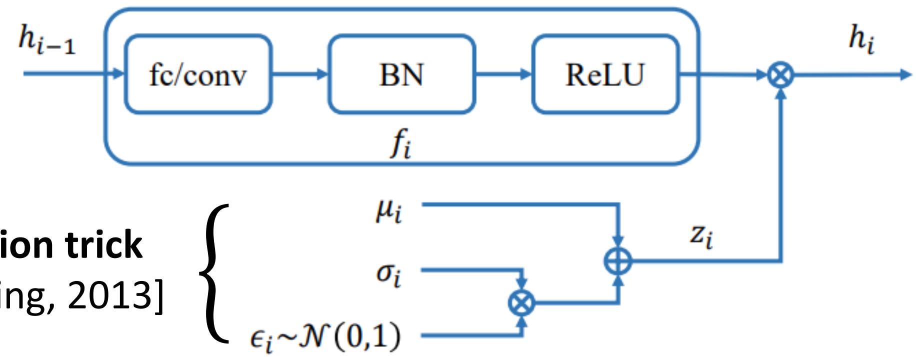
$$\mathcal{L}_i \leq \tilde{\mathcal{L}}_i = \gamma_i \mathbb{E}[\mathrm{KL}(p(\boldsymbol{h}_i|\boldsymbol{h}_{i-1})||q(\boldsymbol{h}_i))] - \mathbb{E}[\log q(\boldsymbol{y}|\boldsymbol{h}_L)]$$

- Final variational objective function (**VIBNet**):

$$\tilde{\mathcal{L}} = \underbrace{\sum_{i=1}^{L} \gamma_i \sum_j \log\left(1 + \boxed{\frac{\mu_{ij}^2}{\sigma_{ij}^2}}\right)}_{\text{regularization}} - \underbrace{\overset{\#\text{ layers}}{L} \cdot \mathbb{E}[\log q(\boldsymbol{y}|\boldsymbol{h}_L)]}_{\text{data-fit}}$$

- **Pruning criteria**: $\alpha_{ij} := \frac{\mu_{ij}^2}{\sigma_{ij}^2} \to 0$
  - Neurons with low value of $\alpha_{ij}$'s are pruned after training

**Reparametrization trick**
[Kingma & Welling, 2013]





*source : Dai et al., "Compressing Neural Networks using the Variational Information Bottleneck", ICML 2018

- **VIBNet** outperforms various methods by large margins
  - $r_W$(%): ratio of # parameters
  - $r_N$(%): ratio of memory footprint

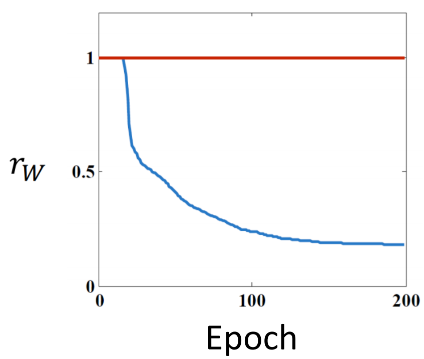
| Method | $r_W(\%)$ | $r_N(\%)$ | error(%) | Pruned Model |
|---|---|---|---|---|
| VD | 25.28 | 58.95 | 1.8 | 512-114-72 |
| BC-GNJ | 10.76 | 32.85 | 1.8 | 278-98-13 |
| BC-GHS | 10.55 | 34.71 | 1.8 | 311-86-14 |
| L0 | 26.02 | 45.02 | **1.4** | 219-214-100 |
| L0-sep | 10.01 | 32.69 | 1.8 | 266-88-33 |
| DN | 23.05 | 57.94 | 1.8 | 542-83-61 |
| VIBNet | **3.59** | **16.98** | 1.6 | **97-71-33** |

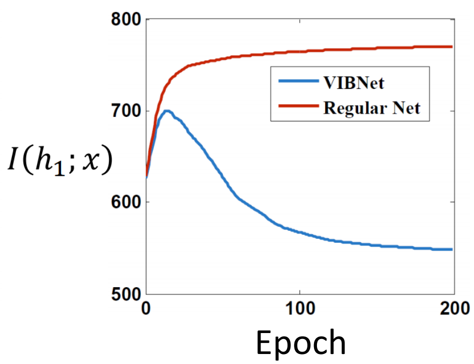
*Table 1.* Compression results on MNIST using LeNet-300-100.

After fine-tuning

| Method | $r_W(\%)$ | FLOP(Mil) | $r_N(\%)$ | error(%) |
|---|---|---|---|---|
| BC-GNJ | 6.57 | 141.5 | 81.68 | 8.6 |
| BC-GHS | 5.40 | 121.9 | 74.82 | 9.0 |
| VIBNet | **5.30** | **70.63** | **49.57** | 8.8 (**8.5**) |
| PF | 35.99 | 206.3 | 83.97 | 6.6 |
| SBP | 7.01 | 136.0 | 80.72 | 7.5 |
| SBPa | 5.78 | 99.20 | 66.46 | 9.0 |
| VIBNet | **5.45** | **86.82** | **57.86** | 6.5 (**6.1**) |
| NS-Single | 11.50 | 195.5 | - | 6.2 |
| NS-Best | 8.60 | 147.0 | - | 5.9 |
| VIBNet | **5.79** | **116.0** | **59.60** | 6.2 (**5.8**) |

*Table 3.* Compression results on CIFAR10 using VGG-16.




| Method | $r_W(\%)$ | FLOP(Mil) | $r_N(\%)$ | error(%) |
|---|---|---|---|---|
| RNP | - | 160 | - | 38.0 |
| VIBNet | **22.75** | **133.6** | **59.80** | 37.6 (**37.4**) |
| NS-Single | 24.90 | 250.5 | - | 26.5 |
| NS-Best | 20.80 | 214.8 | - | 26.0 |
| VIBNet | **15.08** | **203.1** | **73.80** | 25.9 (**25.7**) |

*Table 4.* Compression results on CIFAR100 using VGG-16.



*source : Dai et al., "Compressing Neural Networks using the Variational Information Bottleneck", ICML 2018

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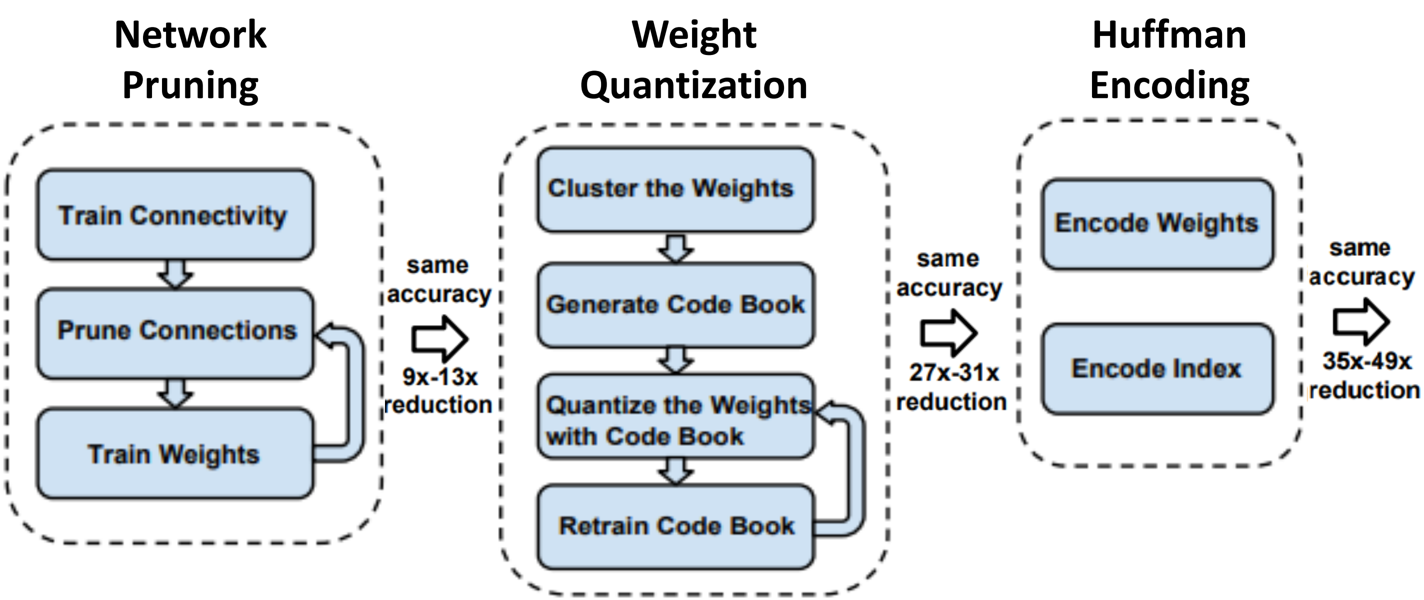
- **Quantizing weights** can further compress the pruned networks
  - Weights are clustered into discrete values
  - The network is represented only with several centroid values

- Han et al. (2015): Pruning DNNs ⇒ 9x-13x reduction
- Han et al. (**2016**): Pruning + Quantization + Huffman ⇒ **35x-49x** reduction



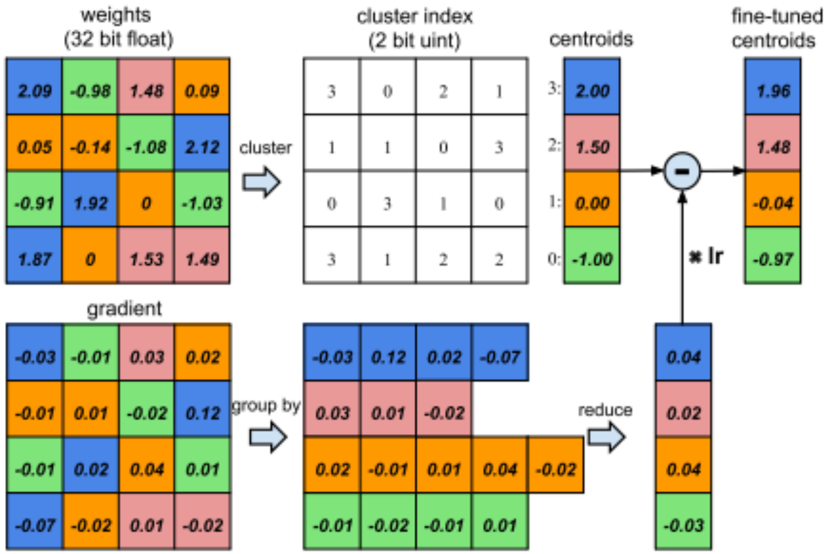

*source : Han et al., "Deep Compression - Compressing Deep Neural Networks with Pruning, Trained Quantization and Huffman Coding", ICLR 2016

- **Quantizing weights** can further compress the pruned networks
  - Weights are clustered into discrete values
  - The network is represented only with several centroid values

1. **Train** a deep model until convergence
2. **Find** $k$ clusters that minimizes **within-cluster sum of squares** (WCSS):
$$\text{argmin}_C \sum_{i=1}^{k} \sum_{w \in c_i} |w - c_i|^2$$
3. **Quantize** with the cluster $C$ via weight sharing
4. **Fine-tune** the network with the shared weights

- In the **fine-tuning** phase, gradients in each cluster are aggregated:

$$\frac{\partial \mathcal{L}}{\partial C_k} = \sum_{i,j} \frac{\partial \mathcal{L}}{\partial W_{ij}} \frac{\partial W_{ij}}{\partial C_k}$$
$$= \sum_{i,j} \frac{\partial \mathcal{L}}{\partial W_{ij}} \mathbf{1}(W_{ij} \in C_k)$$

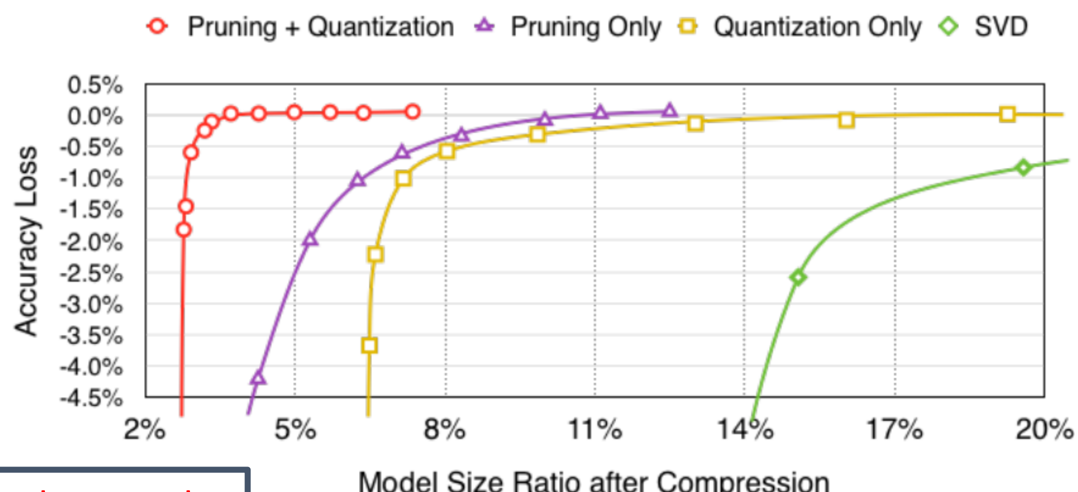



*source : Han et al., "Deep Compression - Compressing Deep Neural Networks with Pruning, Trained Quantization and Huffman Coding", ICLR 2016

# Deep Compression [Han et al., 2016]

- **Deep compression** reduces the model size significantly

| Network | Original Size | Compressed Size | Compression Ratio | Original Accuracy (%) | Compressed Accuracy (%) |
|---|---|---|---|---|---|
| LeNet-300 | 1070KB ⟶ | **27KB** | **40x** | 98.36 ⟶ | 98.42 |
| LeNet-5 | 1720KB ⟶ | **44KB** | **39x** | 99.20 ⟶ | 99.26 |
| AlexNet | 240MB ⟶ | **6.9MB** | **35x** | 80.27 ⟶ | 80.30 |
| VGGNet | 550MB ⟶ | **11.3MB** | **49x** | 88.68 ⟶ | 89.09 |
| GoogLeNet | 28MB ⟶ | **2.8MB** | **10x** | 88.90 ⟶ | 88.92 |
| SqueezeNet | 4.8MB ⟶ | **0.47MB** | **10x** | 80.32 ⟶ | 80.35 |



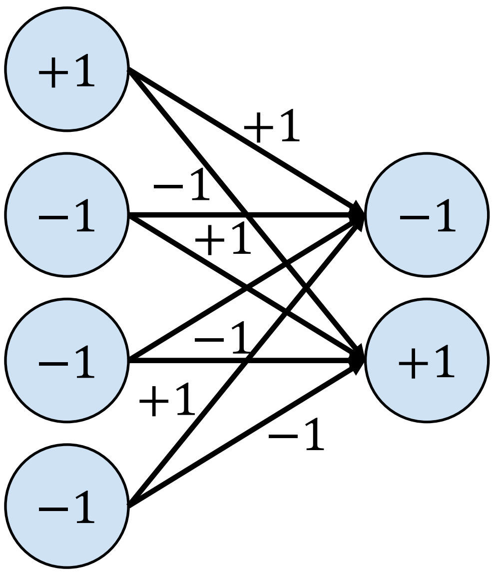

Next: Binarized neural networks

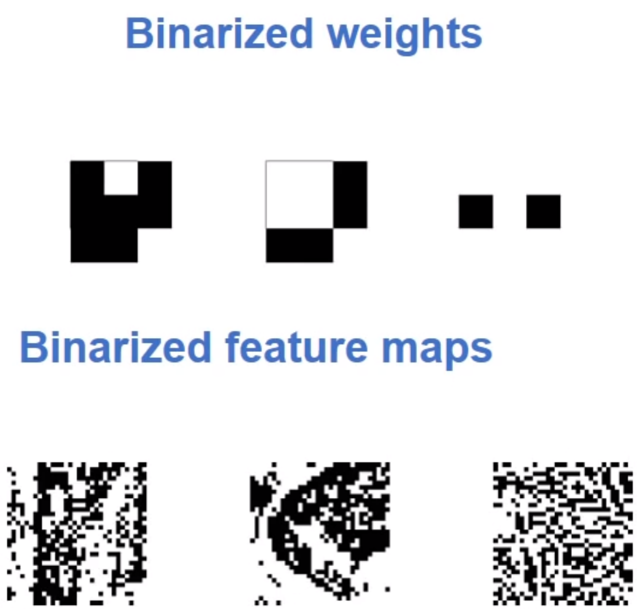
*source : Han et al., "Deep Compression - Compressing Deep Neural Networks with Pruning, Trained Quantization and Huffman Coding", ICLR 2016

- Neural networks can be even **binarized** (**+1** or **-1**)
  - DNNs trained to use binary weights and binary activations

- Expensive **32-bit MAC** (**M**ultiply-**AC**cumulate) ⇒ Cheap **1-bit XNOR-Count**
  - "MAC == XNOR-Count": when the weights and activations are $\pm 1$

# 1s in bits






*source : Hubara et al., "Binarized Neural Networks ", NIPS 2016

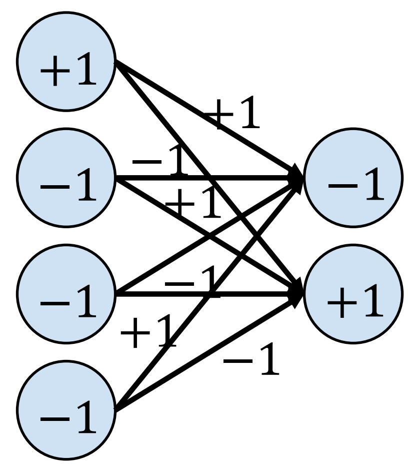
- **Idea**: Training real-valued nets ($W_r$) treating binarization ($W_b$) as noise
  - Training $W_r$ is done by **stochastic gradient descent**

- **Binarization** ($W_r \rightarrow W_b$) occurs for each forward propagation
  - On each of **weights**: $W_b = \mathrm{sign}(W_r)$
  - … also on each **activation**: $a_b = \mathrm{sign}(a_r)$

- Gradients for $W_r$ is estimated from $\frac{\partial L}{\partial W_b}$ [Bengio et al., 2013]
  - "Straight-through estimator": Ignore the binarization during backward!

$$\frac{\partial L}{\partial W_r} = \frac{\partial L}{\partial W_b} \underline{\mathbf{1}_{|W_r| \leq 1}}$$

$$\frac{\partial L}{\partial a_r} = \frac{\partial L}{\partial a_b} \underline{\mathbf{1}_{|a_r| \leq 1}}$$




  - Cancelling gradients for better performance
    - When the value is too large



*source : Hubara et al., "Binarized Neural Networks ", NIPS 2016

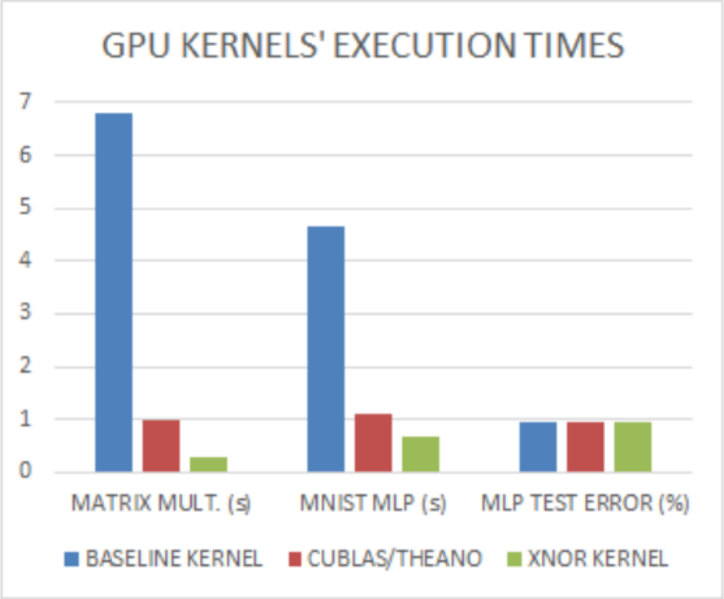
# Binarized Neural Networks [Hubara et al., 2016]

- Neural networks can be even **binarized** (**+1** or **-1**)
  - DNNs trained to use binary weights and binary activations

- BNN yields **32x less memory** compared to the baseline 32-bit DNNs
  - … also expected to reduce energy consumption drastically

- **23x faster** on kernel execution times
  - BNN allows us to use XNOR kernels
  - **3.4x** faster than cuBLAS

| Operation | MUL | ADD |
|---|---|---|
| 8bit Integer | 0.2pJ | 0.03pJ |
| 32bit Integer | 3.1pJ | 0.1pJ |
| 16bit Floating Point | 1.1pJ | 0.4pJ |
| 32tbit Floating Point | 3.7pJ | 0.9pJ |






*source : Hubara et al., "Binarized Neural Networks ", NIPS 2016

# Binarized Neural Networks [Hubara et al., 2016]

- Neural networks can be even **binarized** (**+1** or **-1**)
  - DNNs trained to use binary weights and binary activations

- **BNN** achieves comparable error rates over existing DNNs

| Data set | MNIST | SVHN | CIFAR-10 |
|---|---|---|---|
| Binarized activations+weights, during training and test | | | |
| BNN (Torch7) | 1.40% | 2.53% | 10.15% |
| BNN (Theano) | 0.96% | 2.80% | 11.40% |
| Committee Machines' Array (Baldassi et al., 2015) | 1.35% | - | - |
| Binarized weights, during training and test | | | |
| BinaryConnect (Courbariaux et al., 2015) | $1.29 \pm 0.08\%$ | 2.30% | 9.90% |
| Binarized activations+weights, during test | | | |
| EBP (Cheng et al., 2015) | $2.2 \pm 0.1\%$ | - | - |
| Bitwise DNNs (Kim & Smaragdis, 2016) | 1.33% | - | - |
| Ternary weights, binary activations, during test | | | |
| (Hwang & Sung, 2014) | 1.45% | - | - |
| No binarization (standard results) | | | |
| Maxout Networks (Goodfellow et al.) | 0.94% | 2.47% | 11.68% |
| Network in Network (Lin et al.) | - | 2.35% | 10.41% |
| Gated pooling (Lee et al., 2015) | - | 1.69% | 7.62% |

*source : Hubara et al., "Binarized Neural Networks ", NIPS 2016

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## Summary

- **Broad economic viability requires energy efficient AI** [Welling, 2018]
  - "Energy efficiency of a brain is 100x better than current hardware"
  - "AI algorithms will be measured by the amount of intelligence per kWh"
- **Network pruning and re-wiring**
  - A simple but effective way to compress DNNs
  - Allow us to find better optimum that the current training cannot
- **Sparse network learning**
  - Which training scheme will maximize the pruning performance?
  - It has gained significant attention recently
- **Various other techniques have been also proposed**
  - Weight quantization
  - Anytime/adaptive networks [Huang et al., 2018]
  - …

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