Dynamics in Congestion Games

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Game Theory on Network Science

Game

- Collection of selfish players

Network

- Collection of non-cooperative entities

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- Behavioral rules
  - e.g. best reply, logit-response, fictitious play ...

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Simple behavioral rules have provided insights on

- Engineered networks : Efficient, distributed algorithm design
- Societal networks : Faithful behavioral model

High Level Story of Talk

We present

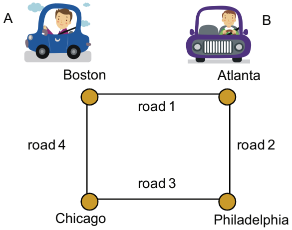
- New learning behavioral rule
  - Simple & Faithful
  - We call it '*logit-response with herd mentality*'.

We study

- Its performance in congestion game
  - Price of anarchy, Convergence rate, Price of dynamics

Outline

Example : Congestion Game



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road 1	2 hours	6 hours
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- Two drivers A and B
  - A wants go from Boston to Philadelphia.
  - B wants go from Atlanta to Chicago.
- They play a game by selecting their own route.

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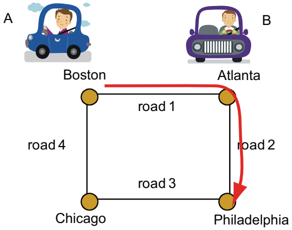

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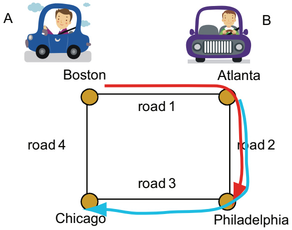

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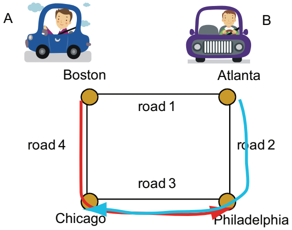

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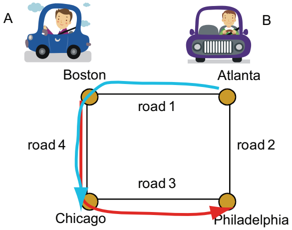

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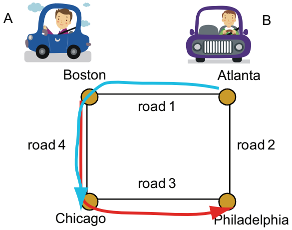

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- Behavior (or Learning) Rule : Best Reply
  - Each driver chooses her best route.
- Nash equilibrium : (A, B)=($4 \to 3$, $1 \to 4$)

Congestion Game

Setup

- $\{1, 2, \dots, n\}$ players (or drivers)
- Each player p has its strategy-set (or routes) R_p
- $u_p(i, \mathbf{s}_{-p})$: utility (or delay) of player p for strategy $i \in R_p$
  - $\mathbf{s}_{-p}$ is the strategy profile of other players other than p

Our interest

- large, varying n and fixed $\cup_p R_p$
- $R_1 = R_2 = \cdots = R_n = \{1, \dots, k\}$
  - i.e. we consider a symmetric setup
  - however, our results/ideas are extendable to a general case

Nash & Price of Anarchy

Nash equilibrium

- The strategy $\mathbf{s} = (s_1, \ldots, s_n)$ is Nash if

$$s_p = \arg\max_i u_p(i, \mathbf{s}_{-p}), \qquad \text{for all } p$$

Price of anarchy in symmetric congestion games

- For linear delay (or utility) functions,

$$\text{PA} = \frac{\text{Total Delay of Nash}}{\text{Optimal Total Delay}} \leq 5/2$$

[Koutsoupias and Papadimitriou 05]

$$\text{BPA} = \frac{\text{Total Delay of Best Nash}}{\text{Optimal Total Delay}} \approx 1$$

if n is large and k is fixed

Modeling Behavior: Rationality

Each player chooses the best strategy

- Best Reply
- i.e. chooses i^* s.t.

$$i^* = \arg\max_i u_p(i, \mathbf{s}_{-p})$$

In congestion games,

- Best Reply always converges to Nash [Monderer and Shapley 96]
  - PA (Price of anarchy) of Best Reply $= 5/2$

Modeling Behavior: Bounded Rationality

However, players can make errors i.e. bounded rationality

- Logit-Response [Blume 1993]: A probabilistic model
- Each player chooses i

$$\text{with probability} \quad \propto \quad \exp[\beta u_p(i, \mathbf{s}_{-p})],$$

where β is a constant to measure the rationality.

Question: converge around Nash?

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Question: converge around Nash?

- Yes, when β is large
  - but in exponential time.
- Further, converge around Best Nash if $\beta < \infty$.
  - PA (Price of anarchy) of Logit-Response $= 1$

Logit-Response: Why Exponential Convergence to Nash?

To study the convergence rate, we need

- Rule for when/who update
  - Logit-Response = Rule for how to update

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  - Each player has an exponential clock of rate 1.
  - When the clock ticks, she update her strategy.

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- Rule for when/who update
  - Logit-Response = Rule for how to update

- Natural assumption : the uniform chance to update among players
  - Each player has an exponential clock of rate 1.
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- Then, Logit-Response = (Continuous time) Markov process on the strategy set
  - Convergence rate = Mixing time

Logit-Response: Why Exponential Convergence to Nash?

Logit-Response = Markov process on the strategy set

- Converge to the stationary distribution π

$$\pi_{\mathbf{s}} \quad \propto \quad \exp[\beta\, P(\mathbf{s})] \times \exp[n\, H(\mathbf{s})], \quad \textit{where} \begin{cases} H = \text{Entropy function} \\ P = \text{Potential function} \\ \text{local (global) maximizer of } P \text{ is (Best) Nash} \end{cases}$$

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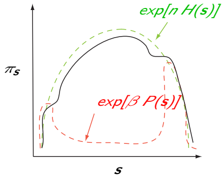
- When $\beta \ll o(n)$
  - π is far from Nash



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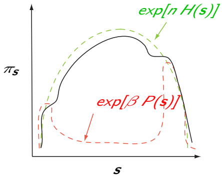
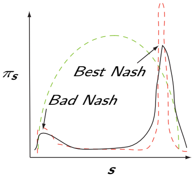


- When $\beta = \Omega(n)$
  - π concentrate on Max of P = Best Nash



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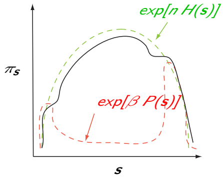

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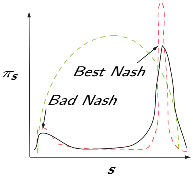

- However, convergence (mixing) time is exponential when $\beta = \Omega(n)$
  - Since possible to be stuck in Bad Nash

Summary of Logit-Response

Pros

- More realistic modeling for societal networks
  - e.g. road networks
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- Something is missing? Possible to fix?

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Herd mentality

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Theorem (Shah and **Shin** 10)

Logit-Response with herd mentality converges around Best Nash in linear time with $\beta = O(1)$.

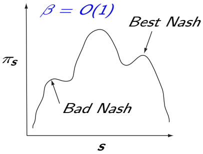
Proof Intuition: Why $\beta = O(1)$?

Recall without herd mentality

- Logit-Response = Markov process with

$$\pi_{\mathbf{s}} \quad \propto \quad \exp[\beta P\ (\mathbf{s})] \times \exp[n\, H(\mathbf{s})]$$

  - $\beta = \Omega(n)$ due to the entropy term $\exp[n\, H(\mathbf{s})]$.



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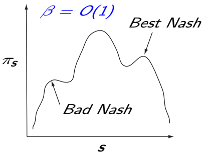

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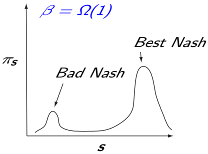

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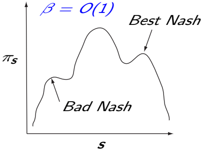

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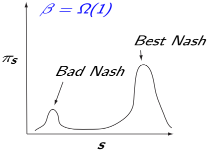

- Key observation: Total clock-rate of players of the strategy i

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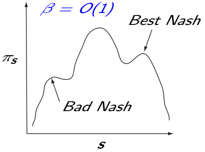
- Clock-rate of player $= 1 \rightarrow$ Choose a player uniformly.



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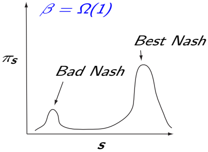

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- We have studied the game under ‘static’ setup.
- Good news
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Question: What is the performance loss due to such dynamics?

- Nash is dynamic.
- Can Logit-Response chase this dynamic Best Nash?

Dynamic Players Setup

Notations

- $n(t)$: the number of players at time t.
  - $n(t) \leftarrow n(t) - 1$ if an old player leaves the game at time t.
  - $n(t) \leftarrow n(t) + 1$ if a new player joins the game at time t.
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  - naturally extendable in this dynamic setup.

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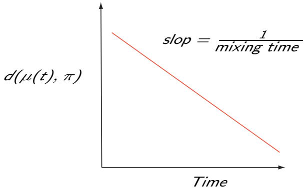
Theorem (Shah and **Shin** 10)

In the dynamic setup, Logit-Response with herd mentality converges around dynamic Best Nash in linear time with $\beta = O(1)$ and $\lambda = O(1)$.

Proof Intuition : Why Converge with $\lambda = O(1)$?

Recall the static setup

- Logit-Response = Markov process
- Stationary distribution $\pi \approx$ Nash
  - $\mu(t)$ denotes the actual distribution



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In the dynamic setup

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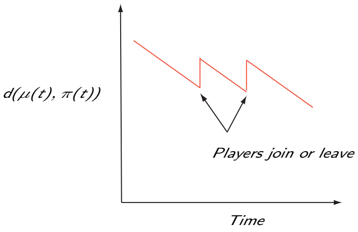

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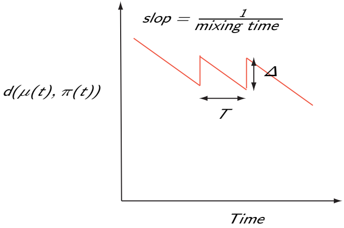

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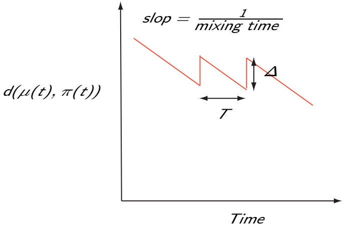

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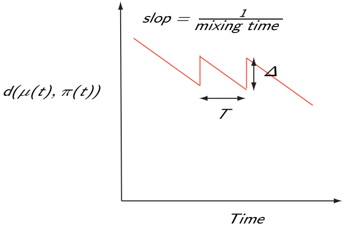

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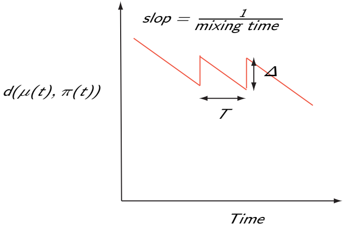

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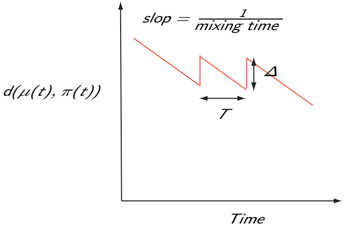

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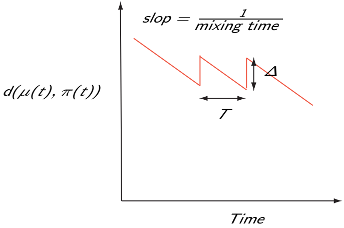

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Δ = How much the mechanism loses mixing when players change

$$d(\mu(t), \pi(t)) \to 0 \quad \text{if} \quad \lambda < \frac{1}{\Delta} \times \frac{1}{\text{mixing time}}$$

d	Δ	mixing time	Condition for λ

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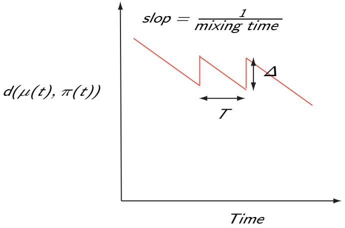

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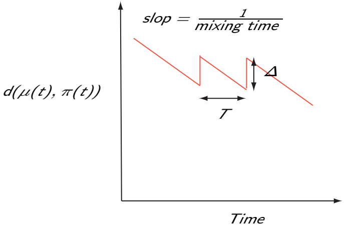

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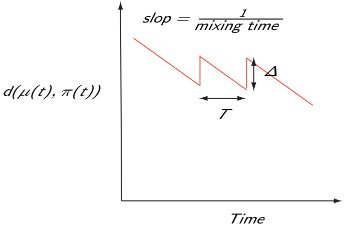

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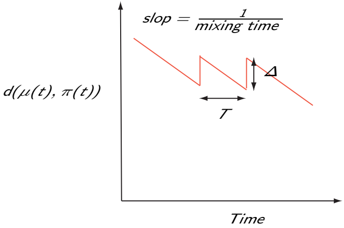

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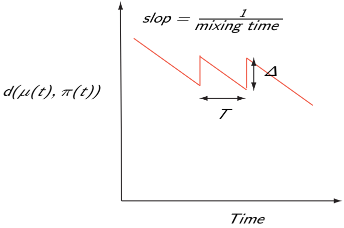

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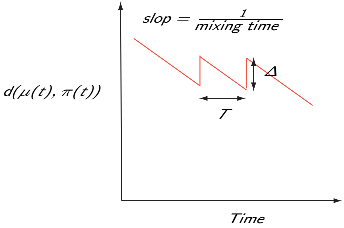

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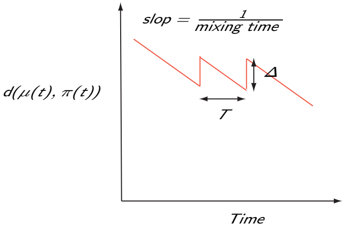

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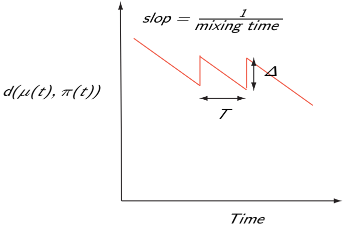

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Will find applications in engineered & societal networks in future