

# Adversarial Example

EE807: Recent Advances in Deep Learning  
Lecture 14

Slide made by

Seokmin Youn and Kimin Lee  
KAIST EE

### **1. Introduction**

- What is the adversarial example?
- Threat of the adversarial example

### **2. Adversarial Attack**

- White-box attack
- Black-box attack

### **3. Adversarial Defense**

- Adversarial training
- Input pre-processing
- Robust network construction

### 1. Introduction

- What is the adversarial example?
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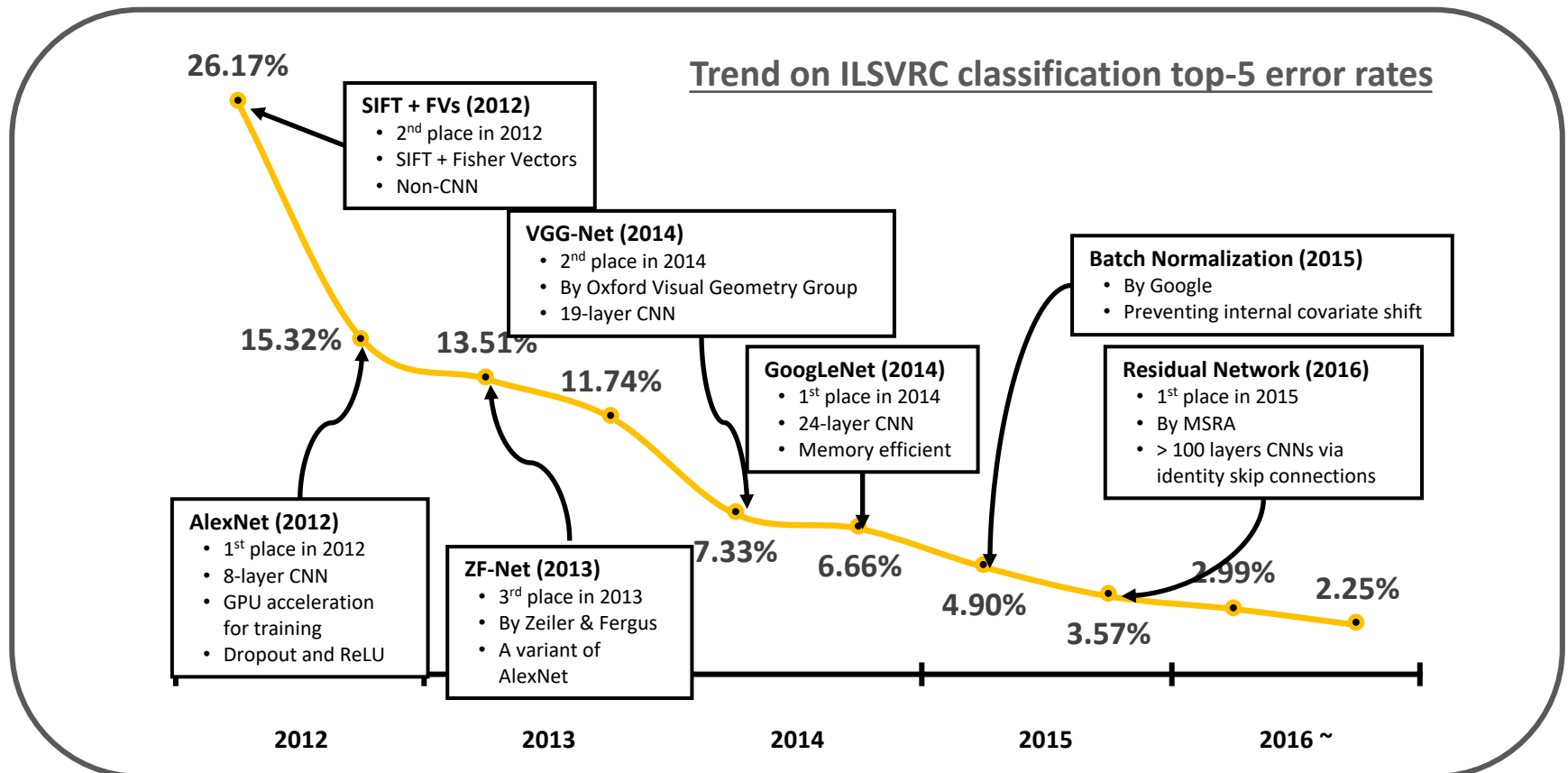
### 2. Adversarial Attack

- White-box attack
- Black-box attack

### 3. Adversarial Defense

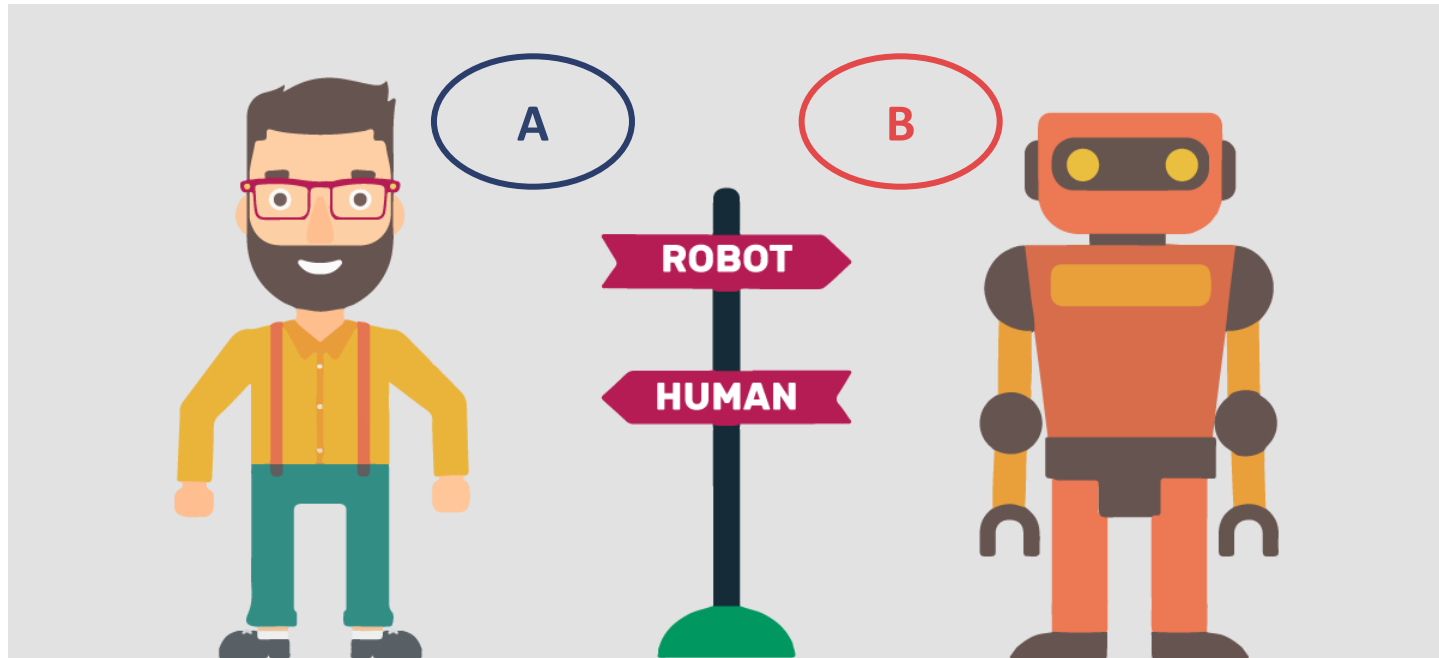
- Adversarial training
- Input pre-processing
- Robust network construction

- Nowadays, **Convolutional Neural Network** shows impressive performance
  - The problem is that a neural network is **highly vulnerable** to a small perturbation of an input



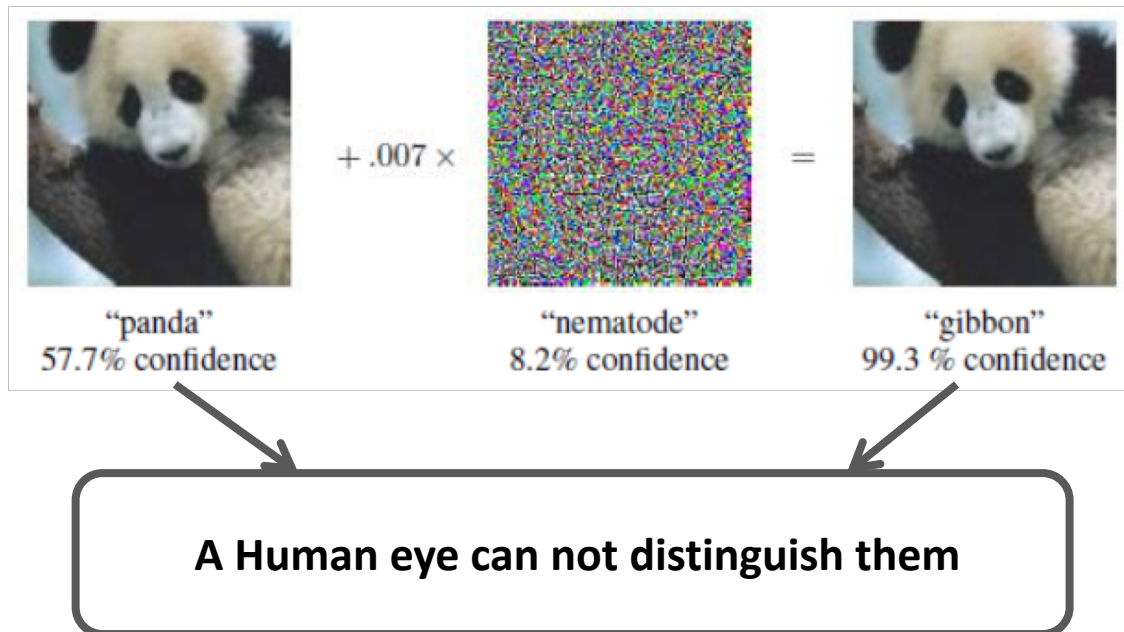


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  - In other words, the **answer of machine** is different from the **answer of human**



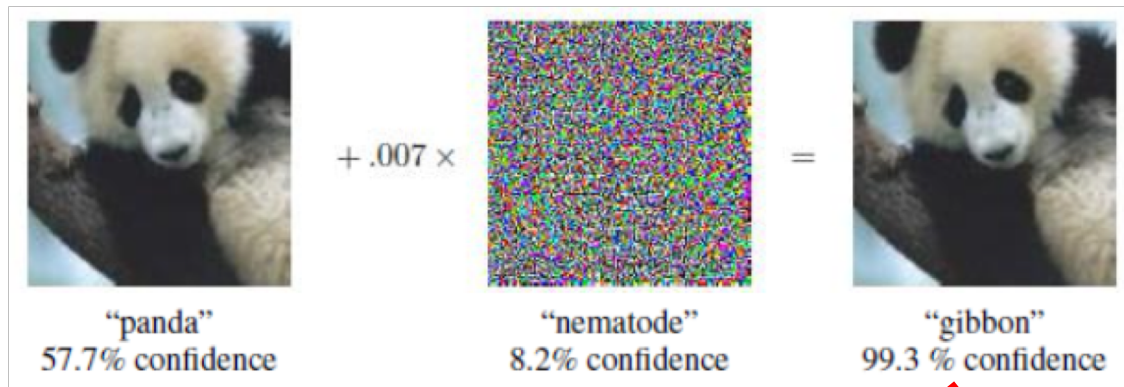
## What is The Adversarial Example?

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- Several machine learning models, including **state-of-the-art** neural networks, **misclassify** examples that are modified from clean data by **imperceptible perturbations**



## What is The Adversarial Example?

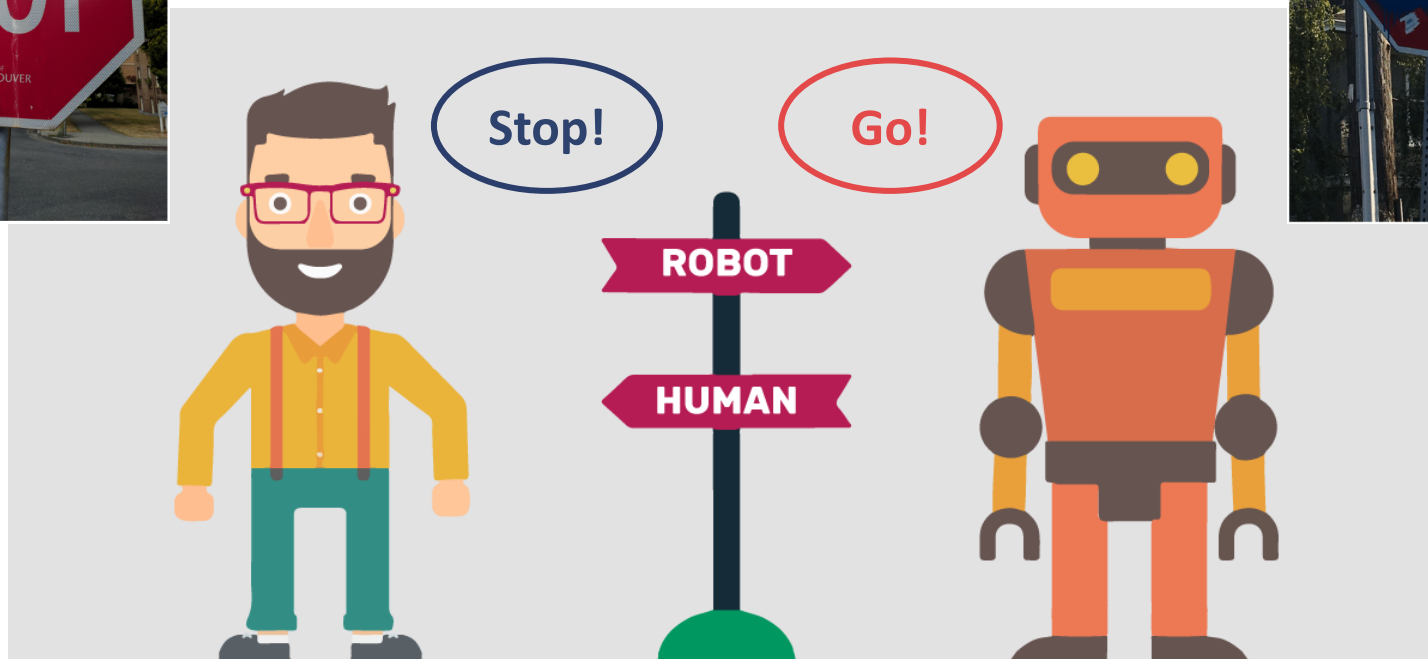
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**It is called an adversarial example!**

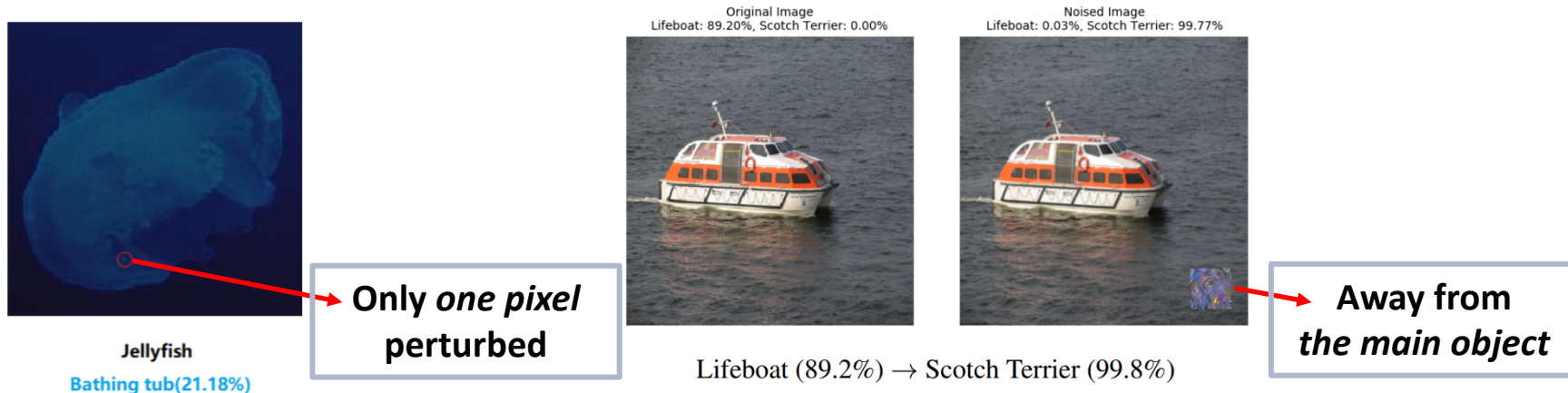
## Threat of The Adversarial Example

- Adversarial examples raise issues that are critical to the safety of AI in the real world
  - e.g. An autonomous vehicle may misclassify graffiti stop signs



# Threat of The Adversarial Example

- There are various types of adversarial perturbations
  - Adversarial perturbations can be constructed in *local regions*



- For segmentation task, adversarial perturbation could control to generate **geometric patterns**



\*source: J. Su et al., One pixel attack for fooling deep neural networks, arXiv, 2017

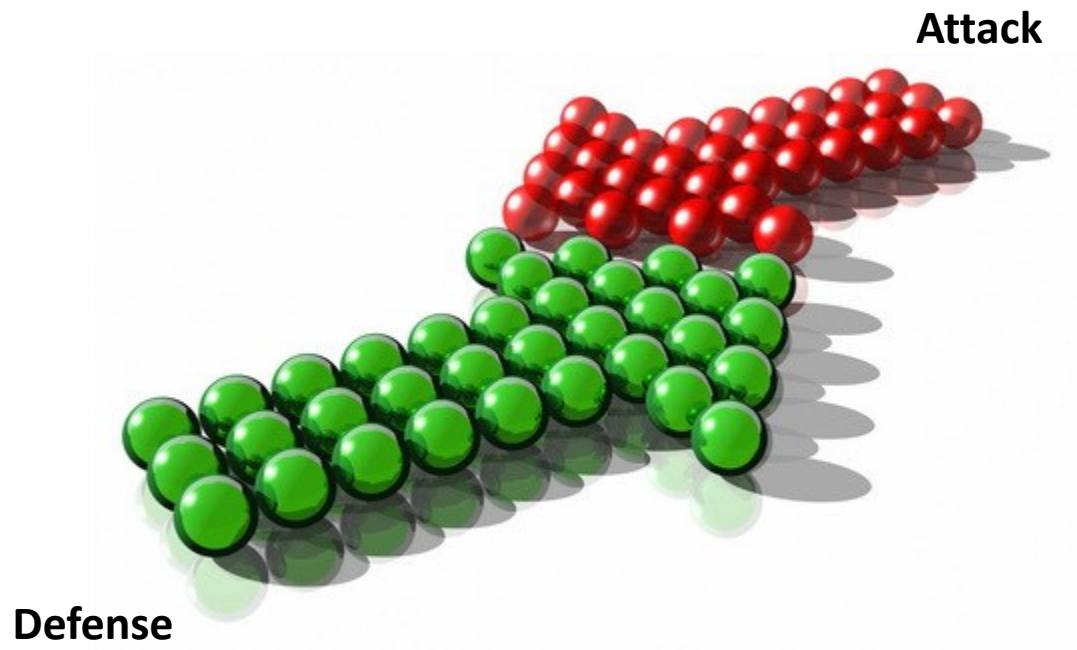
D. Karmon et al., LaVAN: Localized and Visible Adversarial Noise, In ICML, 2018

C. Xie et al., Adversarial Examples for Semantic Segmentation and Object Detection, In ICCV, 2017

## Overview: Studies on The Adversarial Example

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- Studies on the adversarial example are divided into
  - Adversarial **Attack**
    - How to find a perturbation that generate adversarial example
  - Adversarial **Defense**
    - How to prevent a perturbation that generate adversarial example



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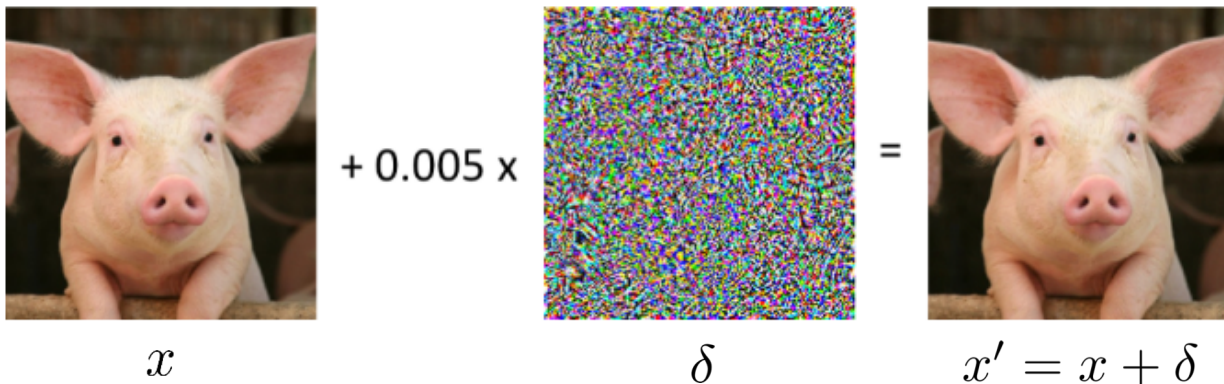
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## Formal Definitions: The Adversarial Example and Attack

- **Definition:** An input  $x'$  is called an **adversarial example** of an input  $x$  if  $x'$  satisfies
  - (1)  $\mathcal{D}(x, x')$  is small for some distance metric  $\mathcal{D}$
  - (2)  $c(x') \neq c^*(x)$  where  $c(\cdot)$  and  $c^*(\cdot)$  denote the prediction and true label
- **Definition: Adversarial attack** is a method of finding adversarial perturbations  $\delta$  that satisfies  $c(x + \delta) \neq c^*(x)$ 
  - The smaller the size of  $\delta$ , the better the adversarial attack method
    - Finding the **smallest** perturbation  $\delta$  is a major challenge





- How to find the adversarial perturbations?
  - **Random perturbation** is the weakest attack method
  - What information is available?
- **White-box Model**
  - Adversary, who creates an adversarial example, has **full knowledge** of the neural network classifiers
  - e.g. model parameters, network architecture, training procedure, ...
- **Black-box Model**
  - Adversary has **no knowledge** of the neural network classifier



white-box



black-box

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- **Motivation:** How to change the network's prediction of an input?
  - In white-box setting, the **gradients** of a network is available
  - **Idea:** A perturbation maximizes the loss function  $\ell(x, y_{\text{true}})$  would change the prediction
  - **Goal:** Solving the objective optimization below by using the linear approximation to  $\ell$

$$\underset{\delta: \|\delta\|_{\infty} \leq \varepsilon}{\text{maximize}} \ell(x + \delta, y_{\text{true}})$$

- **Method: Fast Gradient Sign Method**
  - The adversarial examples are computed by

$$x' = x + \varepsilon \cdot \text{sign}(\nabla_x \ell(x, y_{\text{true}}))$$

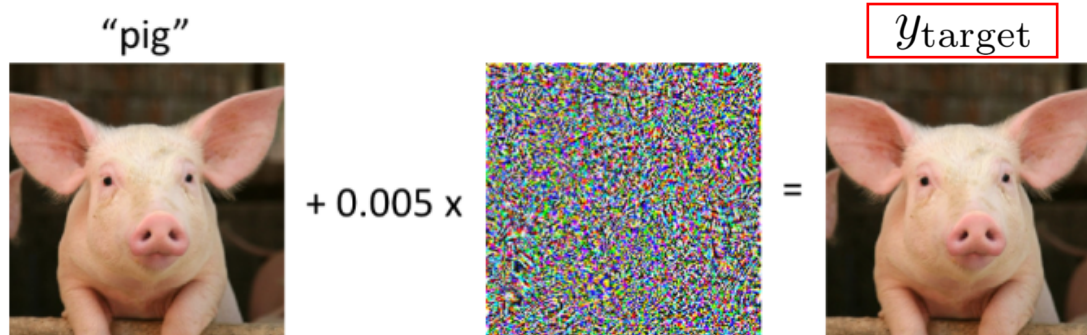
- It can generate adversarial examples fast
  - Generated adversarial examples could have any label (untargeted)

- It is a variant of **Fast Gradient Sign Method**
- **Motivation:** How to **control** the prediction of the adversarial example?
  - **Idea:** A perturbation minimizes the loss function  $\ell(x, y_{\text{target}})$  would change the prediction to the target label
  - **Goal:** Solving the objective optimization below by using the linear approximation to  $\ell$

$$\underset{\delta: \|\delta\|_{\infty} \leq \varepsilon}{\text{minimize}} \ell(x + \delta, y_{\text{target}})$$

- **Method:** The adversarial examples are computed by

$$x' = x - \varepsilon \cdot \text{sign}(\nabla_x \ell(x, y_{\text{target}}))$$



- It is an extension of **Fast Gradient Sign Method**
- **Motivation:** How to find an adversarial perturbation that is stronger than the perturbation from **Fast Gradient Sign Method**?
  - **Idea:** Extending “single-step” to “multi-step”
  - **Goal:** Solving the objective optimization of **Fast Gradient Sign Method** with the number of **iterations**

$$\underset{\delta: \|\delta\|_{\infty} \leq \varepsilon}{\text{maximize}} \ell(x + \delta, y_{\text{true}})$$

- **Method:** The adversarial examples are computed by

$$x_{t+1} = x_t + \alpha \cdot \text{sign}(\nabla_{x_t} \ell(x_t, y_{\text{true}}))$$

$$\text{where } \|x_0 - x_t\|_{\infty} \leq \varepsilon \text{ for all } t$$

- **Least-likely Class Method** also can be extended to **Iterative least-likely class method**

- **Experimental Results**

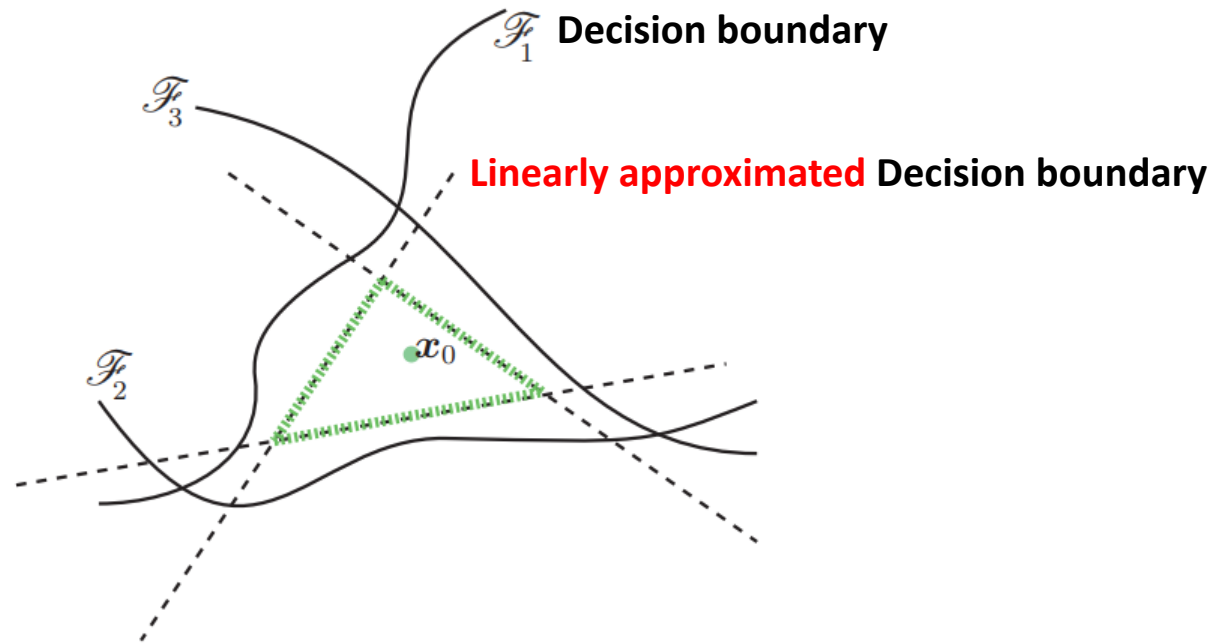
- Comparison between **Fast Gradient Sign**, **Basic Iterative Method**
  - Mean of perturbation and classification accuracy [K. Lee et al., 2018]

		CIFAR-10		CIFAR-100		SVHN	
		$L_\infty$	Acc.	$L_\infty$	Acc.	$L_\infty$	Acc.
DenseNet	Clean	0	95.19%	0	77.63%	0	96.38%
	FGSM	0.21	20.04%	0.21	4.86%	0.21	56.27%
	BIM	0.22	0.00%	0.22	0.02%	0.22	0.67%
ResNet	Clean	0	93.67%	0	78.34%	0	96.68%
	FGSM	0.25	23.98%	0.25	11.67%	0.25	49.33%
	BIM	0.26	0.02%	0.26	0.21%	0.26	2.37%

- **Projected Gradient Method** [A. Madry et al., 2018]

- It is a variant of **Basic Iterative Method**
- **Motivation:** Sometimes, **Basic Iterative Method** falls into local maxima and it does not generate proper adversarial examples
- **Idea:** Adding **random initialization**
- **Method:** Generating a lot of randomly initialized input by adding the random noises before to compute **Basic Iterative Method**

- **Motivation:** Perturbing an input to a **decision boundary** direction would change the prediction of input
  - **Idea:** The smallest adversarial perturbation has a direction to the closest decision boundary
  - **Goal:** Finding a direction to **the closest decision boundary** by using linear approximation to decision boundaries



- **Motivation:** Perturbing an input to a **decision boundary** direction would change the prediction of input
  - **Idea:** The smallest adversarial perturbation has a direction to the closest decision boundary
  - **Goal:** Finding a direction to **the closest decision boundary** by using linear approximation to decision boundaries
  - **Method:** Distance  $d$  from an input  $x$  to a decision boundary between  $y$  and  $y_{\text{true}}$  is computed by

$$d = \frac{|f_y(x) - f_{y_{\text{true}}}(x)|}{\|\nabla_x f_y(x) - \nabla_x f_{y_{\text{true}}}(x)\|_2}$$

where  $f_i(\cdot)$  is a  $i$ -th logit output of the classifier  $f(\cdot)$

- Let  $\hat{d}$  be the smallest distance to decision boundary and  $\hat{y}$ ,  $y_{\text{true}}$  are the corresponding labels. Then the adversarial examples are computed by

$$x_{t+1} = x_t + \hat{d}(f_{\hat{y}}(x_t) - f_{y_{\text{true}}}(x_t))$$



## • Experimental Results

- Comparison between **Fast Gradient Sign, DeepFool Method**
  - Mean of perturbation among four different network

$L_\infty$	MNIST		CIFAR10	
Classifier	LeNet	FC500-150-10	NIN	LeNet
Test acc.	99%	98.3%	88.5%	77.4%
FGSM	0.26	0.11	0.024	0.028
DeeoFool	0.10	0.04	0.008	0.015

- **DeepFool Method** is a stronger method than **Fast Gradient Sign Method**
- Comparison among **Fast Gradient Sign, Basic Iterative, DeepFool Method**
  - Mean of perturbation and classification accuracy [K. Lee et al., 2018]

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	DeepFool	0.30	0.23%	0.25	0.23%	0.57	0.50%
ResNet	Clean	0	93.67%	0	78.34%	0	96.68%
	FGSM	0.25	23.98%	0.25	11.67%	0.25	49.33%
	BIM	0.26	0.02%	0.26	0.21%	0.26	2.37%
	DeepFool	0.36	0.33%	0.27	0.37%	0.62	13.20%

- **DeepFool Method** is not a stronger method than **Basic Iterative Method**

- **Motivation:** Large-scale perturbation should change the prediction, but above attacks are sometimes not successful with large-scale perturbation
  - **Idea:** Finding the smallest perturbation subject to the perturbation makes adversarial example
  - **Goal:** Minimizing the scale of adversarial perturbation  $\|\delta\|$  subject to the perturbation  $\delta$  makes the input to be an adversarial example

$$\underset{\delta: c(x+\delta)=y_{\text{target}}}{\text{minimize}} \quad \|\delta\|_2$$

- **Method:** Applying the **Lagrangian relaxation** to the objective with a function  $g$  that satisfying

$$g(x) = \left( \max_{i \neq \text{target}} f_i(x) - f_{y_{\text{target}}}(x) \right)^+$$

where  $f_i(\cdot)$  is a  $i$ -th logit output of the given classifier  $f(\cdot)$   
and  $(e)^+ = \max(e, 0)$

- $g(x)$  has the minimum value 0 when  $x$  is the adversarial example

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$$\underset{\delta: c(x+\delta)=y_{\text{target}}}{\text{minimize}} \quad \|\delta\|_2$$

- **Method:** Solving the relaxed objective

$$\underset{\delta}{\text{minimize}} \quad \|\delta\|_2 + \alpha \cdot g(x + \delta)$$

$\alpha$ : hyper-parameter

- Use Gradient Descent to solve the optimization

- **Experimental Results**

- Comparison between among **Fast Gradient Sign, Basic Iterative, DeepFool, Carlini-Wagner Method**

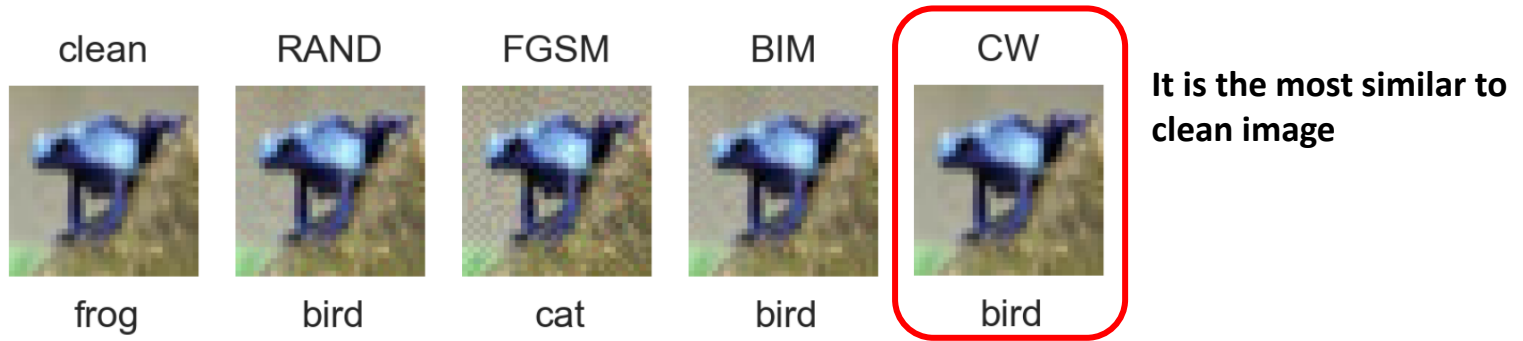
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	DeepFool	0.30	0.23%	0.25	0.23%	0.57	0.50%
	CW	0.05	0.10%	0.03	0.16%	0.12	0.54%
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	BIM	0.26	0.02%	0.26	0.21%	0.26	2.37%
	DeepFool	0.36	0.33%	0.27	0.37%	0.62	13.20%
	CW	0.08	0.00%	0.08	0.01%	0.15	0.04%

- **Carlini-Wagner Method** find the smallest adversarial perturbations among the several attacks
- 10× slower than **Basic Iterative Method**

- **Experimental Results**

- Comparison between among **Fast Gradient Sign, Basic Iterative, DeepFool, Carlini-Wagner Method**
  - Mean of perturbation and classification accuracy [K. Lee et al., 2018]
- Images of adversarial examples among the several attacks [Y. Song et al., 2018]



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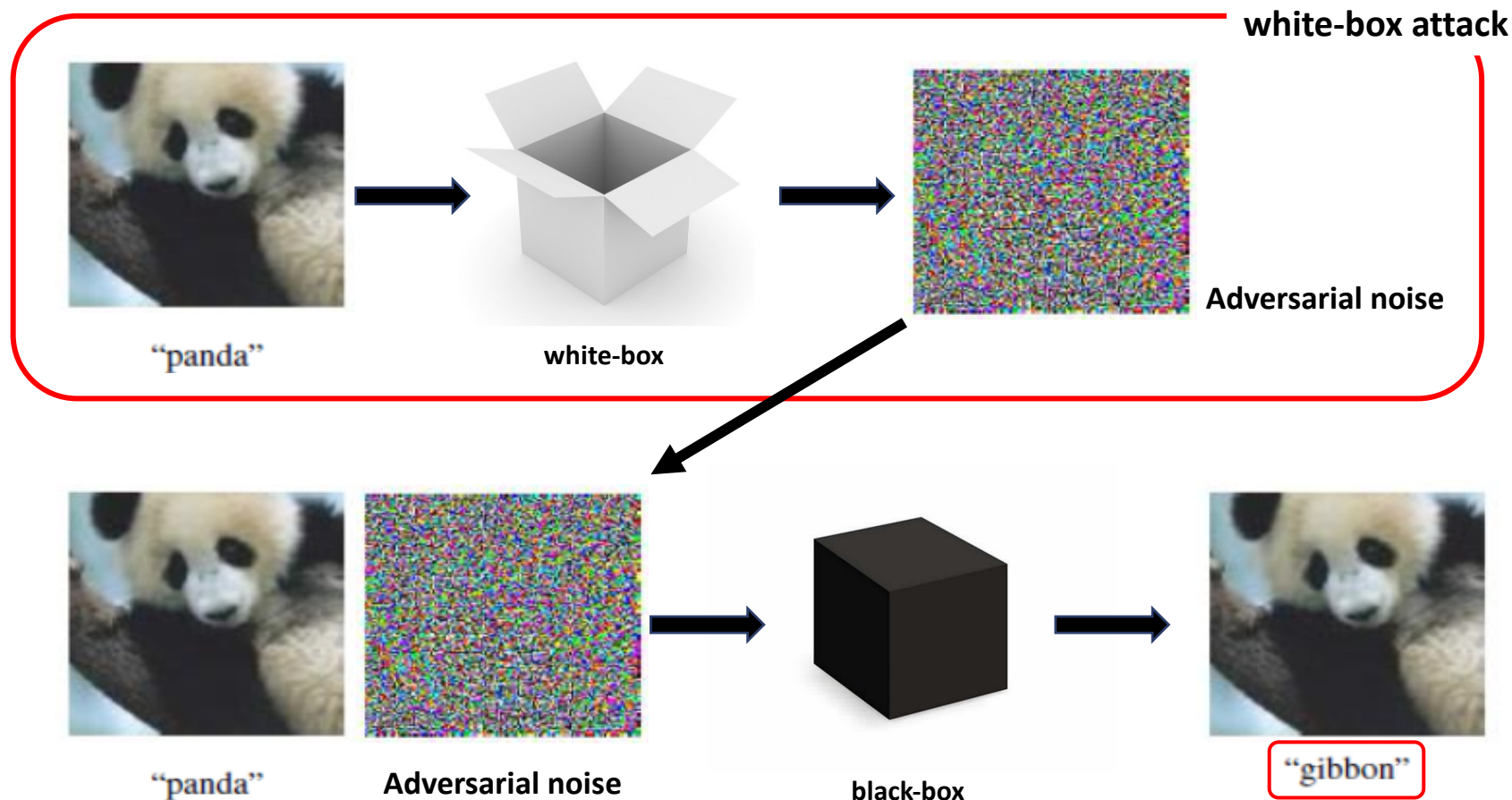
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## Black-Box: Transferability of Adversarial Example

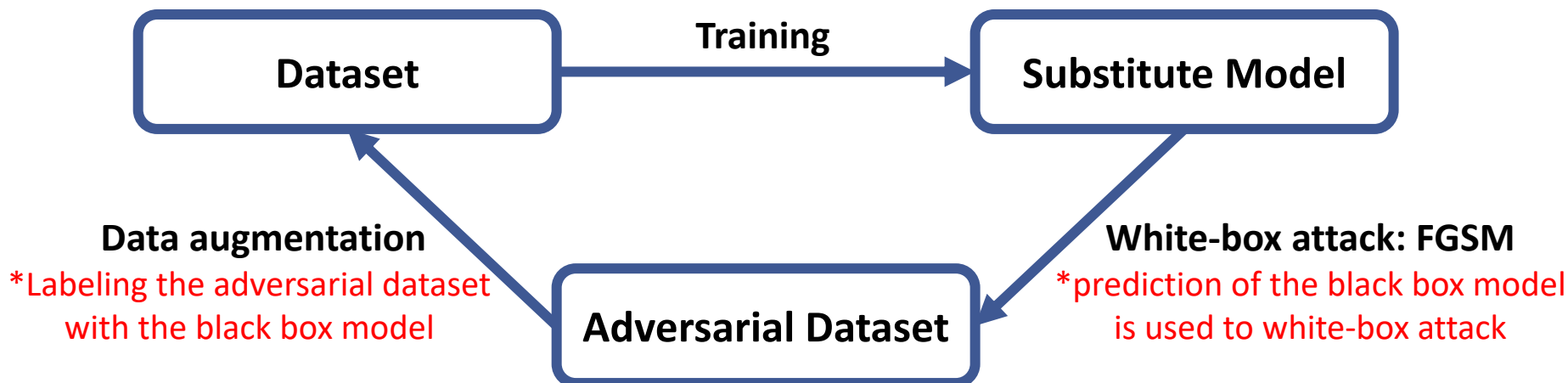
- Some of perturbations make adversarial example among **different network architecture**



- **Motivation:** The transferability of adversarial example allow to attack a black-box model
  - **Idea:** Finding an adversarial example via white-box attack on the local substitute model
  - **Goal:** Training a **local substitute model** via FGSM-based adversarial dataset augmentation
    - FGSM-based adversarial examples are computed to change the prediction of the black-box model

$$x' = x + \varepsilon \cdot \text{sign}(\nabla_x \ell(x, y_{\text{pred}}))$$

- **Method:**





- **Experimental Results**

- The local substitute model training
  - Initial training dataset: subset of **MNIST**, **Handcrafted** set
  - **Handcrafted** set is used to ensure the results do not stem from the similarities between the MNIST test and training sets



- Accuracies of the local substitute models

Substitute Epoch	Initial Substitute Training Set from	
	MNIST test set	Handcrafted digits
0	24.86%	18.70%
1	41.37%	19.89%
2	65.38%	29.79%
3	74.86%	36.87%
4	80.36%	40.64%
5	79.18%	56.95%
6	81.20%	67.00%

- **Experimental Results**

- Black-box attack to the **Amazon** and **Google** Oracle
- Two types of architecture:
  - **DNN**: Deep Neural Network
  - **LR**: Logistic Regression

**Misclassification rates (%)**

Epochs	Queries	Amazon		Google	
		DNN	LR	DNN	LR
$\rho = 3$	800	87.44	96.19	84.50	88.94
$\rho = 6$	6,400	<b>96.78</b>	<b>96.43</b>	<b>97.17</b>	92.05

Number of queries to train the local substitute model

- **Motivation:** Adversarial examples from the substitute model are sometimes weak
  - **Idea:** White-box attack to an **ensemble** of the substitute models could generate the strong adversarial example
  - **Goal:** Finding the adversarial examples from the **ensemble** model
  - **Method:** Consider  $k$  number of substitute models and let  $J_1, \dots, J_k$  be their softmax outputs. Then for given  $(x, y_{\text{true}})$ , the objective is follow:

$$\underset{\delta}{\text{minimize}} \quad -\log \left( 1 - \left( \sum_{i=1}^k \alpha_i J_i(x + \delta) \right)_{\boxed{y_{\text{true}}}} \right) + \lambda d(x, x + \delta)$$

$y_{\text{true}}$ -th softmax output of the ensemble model

where  $\alpha_i$  is a ensemble weight with  $\sum_{i=1}^k \alpha_i = 1$ ,

$$d(x, x') = \sqrt{(\sum_i (x'_i - x_i)^2 / N)}, \quad x, x' \in \mathbb{R}^N$$

$\lambda$ : hyper-parameter

- The metric  $d(x, x')$  is called the Root Mean Square Deviation (RMSD)

- **Experimental Results**

- Ensemble of modern architecture DNNs
  - “-model A” means an ensemble without “model A”
- RMSD: Root Mean Square Deviation of adversarial perturbations







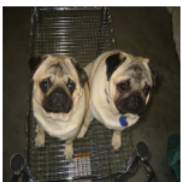
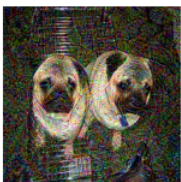
Black-box models

	RMSD	ResNet-152	ResNet-101	ResNet-50	VGG-16	GoogLeNet
-ResNet-152	17.17	0%	0%	0%	0%	0%
-ResNet-101	17.25	0%	1%	0%	0%	0%
-ResNet-50	17.25	0%	0%	2%	0%	0%
-VGG-16	17.80	0%	0%	0%	6%	0%
-GoogLeNet	17.41	0%	0%	0%	0%	5%

Adversarial examples from the ensemble models via white-box attack

- Experimental Results

- Black-box attack on Clarifai.com (commercial black-box image classification system)

original image	true label	Clarifai.com results of original image	target label	targeted adversarial example	Clarifai.com results of targeted adversarial example
	viaduct	bridge, sight, arch, river, sky	window screen		window, wall, old, decoration, design
	hip, rose hip, rosehip	fruit, fall, food, little, wildlife	stupa, tope		Buddha, gold, temple, celebration, artistic
	dogsled, dog sled, dog sleigh	group together, four, sledge, sled, enjoyment	hip, rose hip, rosehip		cherry, branch, fruit, food, season
	pug, pug-dog	pug, friendship, adorable, purebred, sit	sea lion		sea seal, ocean, head, sea, cute

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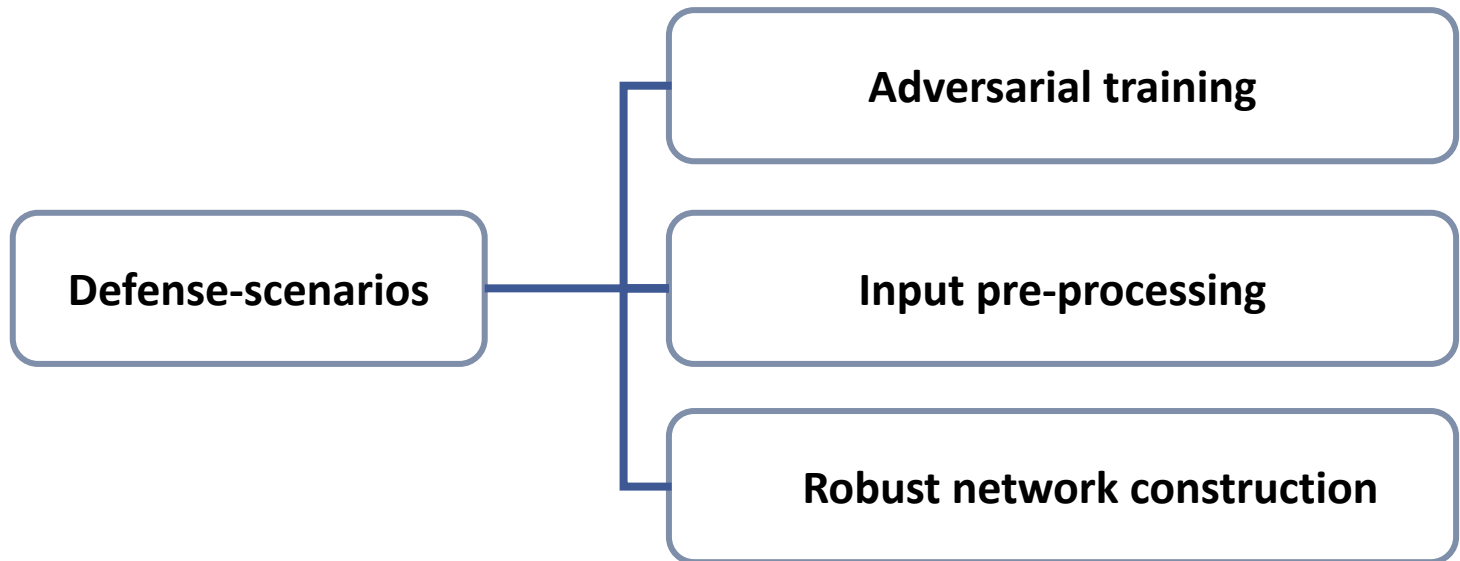
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- How to defend such adversarial attack?
  - 3 types of **Defense scenarios**
    - **Adversarial training**: Re-train with adversarial examples
    - **Input pre-processing**: Pre-process an input to make a clean example
    - **Robust network construction**: Construct a new network to be robust



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- **Motivation:** Training a model with adversarial examples would make the model to be robust on adversarial examples
  - **Idea:** Re-training a model with adversarial examples which are combined to true labels
  - **Goal:** Solving the min-max objective optimization below

$$\underset{\theta \in \Theta}{\text{minimize}} \mathbb{E}_{p(x,y)} \left[ \max_{\delta: \|\delta\|_{\infty} \leq \varepsilon} \ell(\theta; x + \delta, y) \right]$$

- Adversarial examples could substitutes the **inner-maximization** of the objective
- **Question:** How to generate adversarial examples?
  - **FGSM:** Fast and simple (single-step) method
  - **BIM:** Slow, but strong (multi-step) method
  - **Carlini-Wagner Method:** Extremely slow to use adversarial training
    - Not appropriate to adversarial training

- **Method:** Generate adversarial examples via **FGSM** during the training
  - Then total loss function is

$$\widehat{\ell}(\theta; x, y) = \alpha \ell(\theta; x, y) + (1 - \alpha) \ell(\theta; x + \varepsilon \text{sign}(\nabla_x \ell(\theta; x, y)), y)$$

$\alpha$ : hyper-parameter

FGSM example

### Experimental Results

- Robust in FGSM-attack, but not in BIM-attack [A. Kurakin et al., 2017b]

FGSM		Clean	$\varepsilon = 2$	$\varepsilon = 4$	$\varepsilon = 8$	$\varepsilon = 16$
Standard training	top 1	78.4%	30.8%	27.2%	27.2%	29.5%
	top 5	94.0%	60.0%	55.6%	55.6%	57.2%
Adv. training	top 1	77.6%	73.5%	74.0%	74.5%	73.9%
	top 5	93.8%	91.7%	91.9%	92.0%	91.4%

BIM		Clean	$\varepsilon = 2$	$\varepsilon = 4$	$\varepsilon = 8$	$\varepsilon = 16$
Standard training	top 1	77.4%	29.1%	7.5%	3.0%	1.5%
	top 5	93.9%	56.9%	21.3%	9.4%	5.5%
Adv. training	top 1	78.3%	23.3%	5.5%	1.8%	0.7%
	top 5	94.1%	49.3%	18.8%	7.8%	4.4%

noise scale  $\varepsilon \in [0, 255]$

- **Motivation:** Adversarial training against a strong adversarial attacks (e.g. **BIM**) makes the network be a constant
  - It sacrifices the classification performance
  - **Idea:** Large capacity network would not sacrifice the classification performance
  - **Goal:** Solving the min-max objective optimization below

$$\underset{\theta \in \Theta}{\text{minimize}} \mathbb{E}_{p(x,y)} \left[ \max_{\delta: \|\delta\|_{\infty} \leq \varepsilon} \ell(\theta; x + \delta, y) \right]$$

- Substituting the **inner-maximization** of the objective via strong adversarial attack (**PGD**)
- **Method:** Generate adversarial examples via **PGD** during the training
  - Then total loss function is

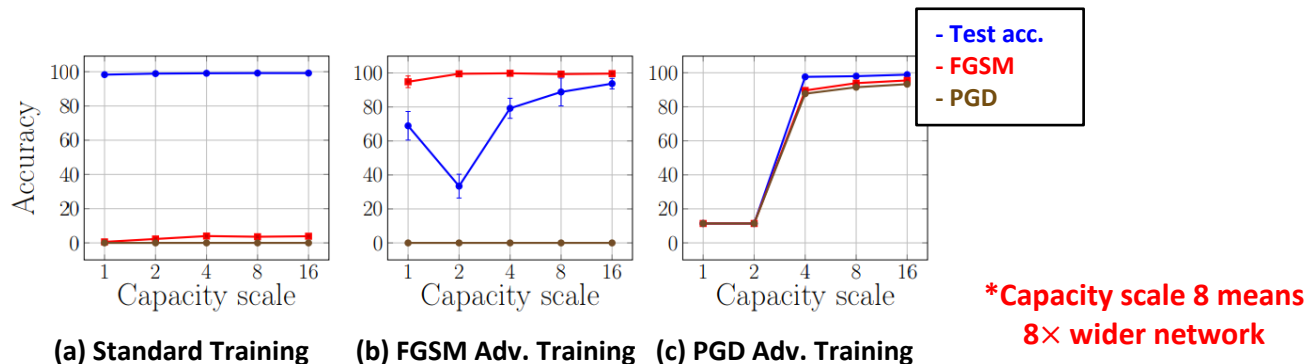
$$\widehat{\ell}(\theta; x, y) = \alpha \ell(\theta; x, y) + (1 - \alpha) \ell(\theta; x', y)$$

$\alpha$ : hyper-parameter

 **PGD example**

## • Experimental Results

- PGD adversarial training with **small** and **large capacity**
- MNIST



- CIFAR10

	(a) Standard Training		(b) FGSM Adv. Training		(c) PGD Adv. Training	
	Simple	Wide	Simple	Wide	Simple	Wide
<b>Test acc.</b>	92.7%	95.2%	87.4%	90.3%	79.4%	87.3%
<b>FGSM</b>	27.5%	32.7%	90.9%	95.1%	51.7%	56.1%
<b>PGD</b>	0.8%	3.5%	0.0%	0.0%	43.7%	45.85%

\*Simple: ResNet

Wide: ResNet with 10× wider

- **Motivation:** Adversarial examples substitute the inner-maximization objective of adversarial training,

$$\underset{\theta \in \Theta}{\text{minimize}} \mathbb{E}_{p(x,y)} \left[ \max_{\delta: \|\delta\|_{\infty} \leq \varepsilon} \ell(\theta; x + \delta, y) \right]$$

- But, the **gap** between the **worst case** (inner-maximization objective) and the adversarial example depends on adversarial attack methods
- It can not be ensured that white-box attacks converges to the **worst case**
  - Inner-maximization objective is generally non-concave
- **Idea:** Making a concave objective by applying the **Lagrangian relaxation** to the inner-maximization objective
- The adversarial-training objective is newly defined as

$$\underset{\theta \in \Theta}{\text{minimize}} \sup_{P \in \mathcal{P}} \mathbb{E}_P [\ell(\theta; Z)]$$

where  $Z = (X, Y) \sim P_0$  with the training data  $X$  and the label  $Y$   
and  $\mathcal{P}$  is a class of distribution around the data-generating distribution  $P_0$

- $\mathcal{P}$  can be written as the **Wasserstein** metric  $W_c$

$$\mathcal{P} = \{P : W_c(P, P_0) \leq \rho\} \quad \text{where a hyper-parameter } \rho \geq 0$$

- Wasserstein metric  $W_c$  between distributions  $P$  and  $Q$  is defined as

$$W_c(P, Q) := \inf_{M \in \Pi(P, Q)} \mathbb{E}_M [c(Z, Z')]$$

where  $c(z, z') := \|x - x'\|_p^2 + \infty \cdot 1_{y \neq y'}$ ,  $1_y$  is a indicator function

- The objective cover the worst-case with some  $P' \in W_c(P', P_0) \leq \rho$

$$\underset{\theta \in \Theta}{\text{minimize}} \quad \sup_{P \in W_c(P, P_0) \leq \rho} \mathbb{E}_P [\ell(\theta; Z)]$$

- **Lagrangian relaxation** for a fixed parameter  $\gamma$  induces the objective to

$$\longrightarrow \underset{\theta \in \Theta}{\text{minimize}} \sup_P \mathbb{E}_P [\ell(\theta; Z) - \gamma W_c(P, P_0)]$$

- Furthermore, theorem [J. Blanchet et al., 2016] induces the **relaxed objective** to

$$\longrightarrow \underset{\theta \in \Theta}{\text{minimize}} \mathbb{E}_{P_0} \left[ \sup_{z \in \mathcal{Z}} \{ \ell(\theta; z) - \gamma c(z, z_0) \} \right]$$

- It is the **final objective** of Wasserstein adversarial training [A. Sinha et al., 2018]
- Goal:** Solving the **final objective** optimization below

$$\underset{\theta \in \Theta}{\text{minimize}} \mathbb{E}_{P_0} \left[ \sup_{z \in \mathcal{Z}} \{ \ell(\theta; z) - \gamma c(z, z_0) \} \right]$$

where  $c(z, z') := \|x - x'\|_p^2 + \infty \cdot 1_{y \neq y'}$ ,  $1_y$  is a indicator function

- The objective of previous adversarial training is follow:

$$\underset{\theta \in \Theta}{\text{minimize}} \mathbb{E}_{p(x, y)} \left[ \max_{\delta: \|\delta\|_\infty \leq \varepsilon} \ell(\theta; x + \delta, y) \right]$$

- **Method:** Wasserstein adversarial training algorithm

---

**Algorithm 1** Distributionally robust optimization with adversarial training

---

INPUT: Sampling distribution  $P_0$ , constraint sets  $\Theta$  and  $\mathcal{Z}$ , stepsize sequence  $\{\alpha_t > 0\}_{t=0}^{T-1}$

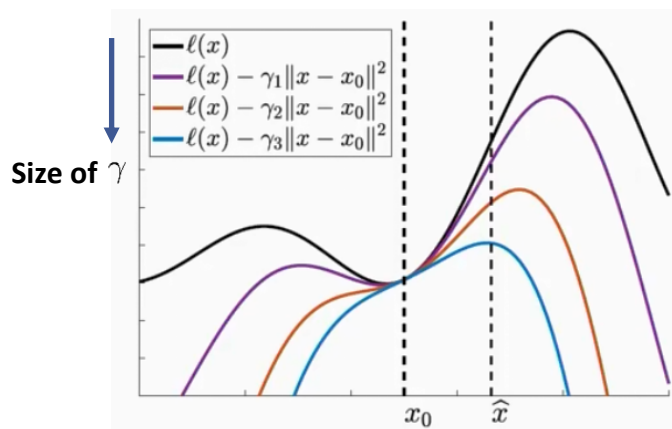
**for**  $t = 0, \dots, T - 1$  **do**

    Sample  $z^t \sim P_0$  and find an  $\epsilon$ -approximate maximizer  $\hat{z}^t$  of  $\ell(\theta^t; z) - \gamma c(z, z^t)$

$\theta^{t+1} \leftarrow \text{Proj}_{\Theta}(\theta^t - \alpha_t \nabla_{\theta} \ell(\theta^t; \hat{z}^t))$

---

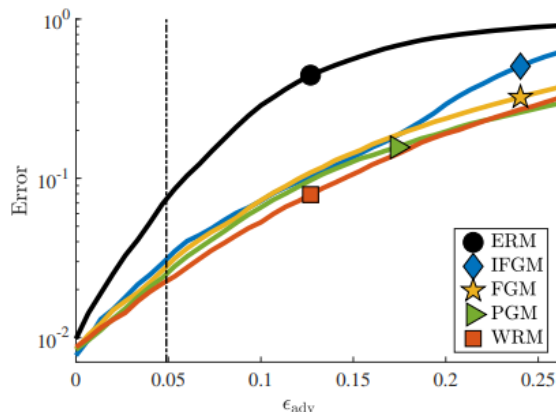
- Large enough  $\gamma$  makes  $\ell(\theta^t; z) - \gamma c(z, z^t)$  concave, and it helps the optimization be easy



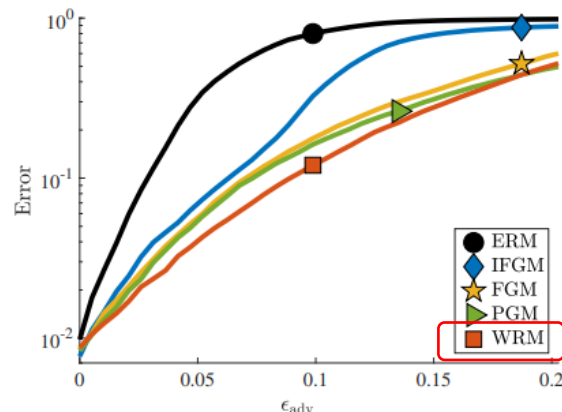
- The algorithm is **attack-agnostic**
  - It does not need any adversarial attack method



- **Experimental Results:** white-box attack with  $l_2$  and  $l_\infty$  metric
  - It shows Wasserstein adversarial training (**WRM**) outperform the baselines (other adversarial trainings)



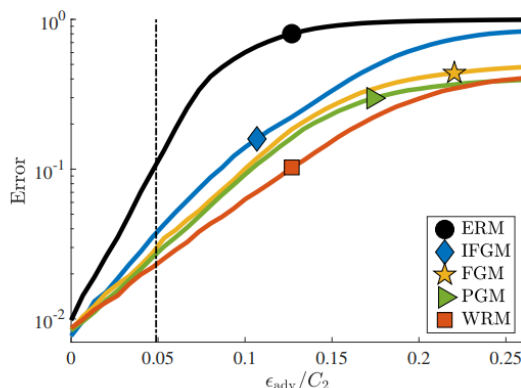
(a) Test error vs.  $\epsilon_{adv}$  for  $\| \cdot \|_2$ -FGM attack



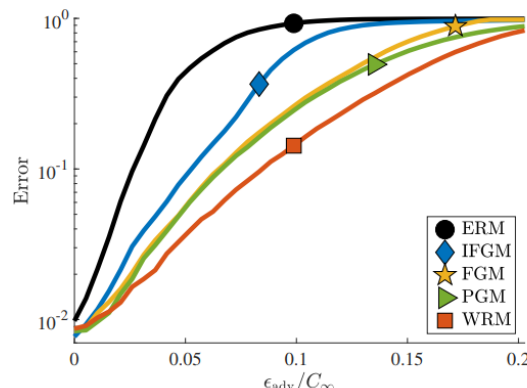
(b) Test error vs.  $\epsilon_{adv}$  for  $\| \cdot \|_\infty$ -FGM attack

**Best performance among the baselines**

- \*ERM: Standard Training
- \*IFGM: BIM Adv. Training
- \*FGM: FGSM Adv. Training
- \*PGM: PGD Adv. Training
- \*WRM: Wasserstein Adv. Training



(a) Test error vs.  $\epsilon_{adv}$  for  $\| \cdot \|_2$  **attack**



(b) Test error vs.  $\epsilon_{adv}$  for  $\| \cdot \|_\infty$  **attack**

**\* PGD attack**

**\* PGD attack**

## 1. Introduction

- What is adversarial example?
- Overview

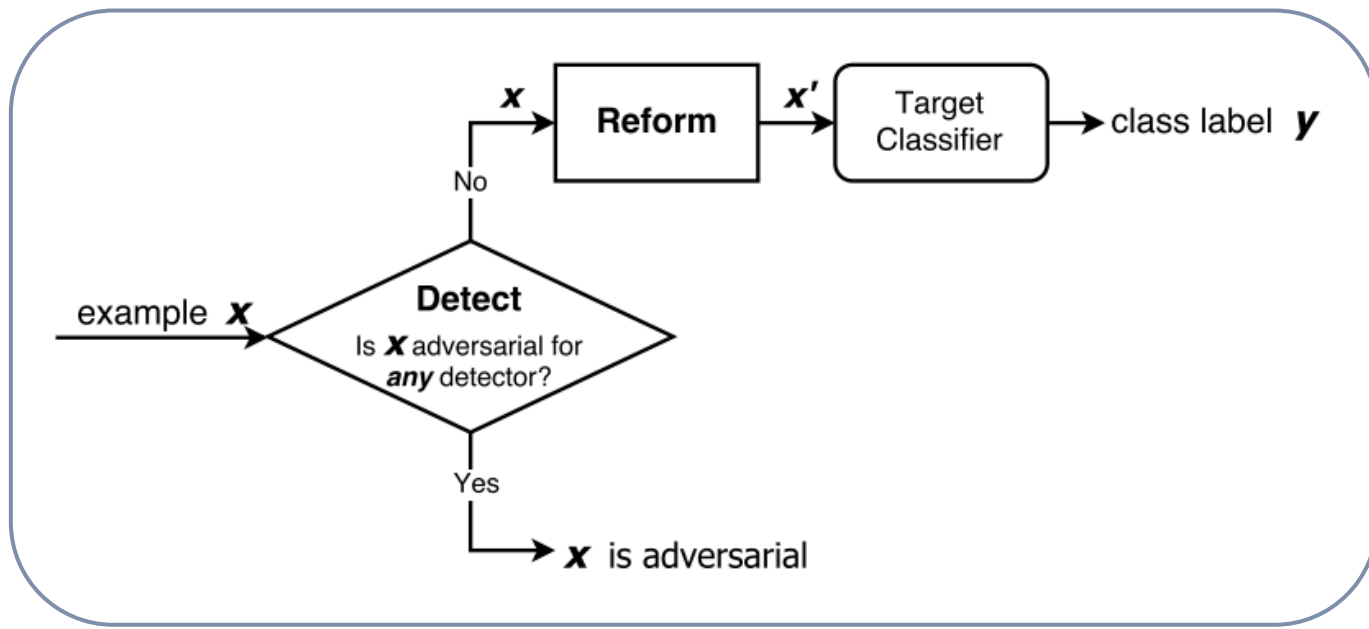
## 2. Adversarial attack

- White-box attack
- Black-box attack

## 3. Adversarial Defense

- Adversarial training
- Input pre-processing
- Robust network construction

- **Motivation:** Pre-process a input to make a clean data
  - **Denoise** adversarial perturbation
  - **Idea:** Double-check an input with **Detector** and **Reformer**
    - Detecting an input whether it is an adversarial example or not
    - Reforming the input which is detected as clean one to prepare in case of the **Detector's** failure



- **Goal:** Training **Detector** and **Reformer** which are based on auto-encoders
- **Method:**
  - **Detector:** Detect an input  $x$  is an adversarial example or not
    - Training an auto-encoder  $f_{\text{AE}}$  to minimize the loss over the training data  $X$

$$\ell(X) = \frac{1}{|X|} \sum_{x \in X} \|x - f_{\text{AE}}(x)\|_2$$

- **Detector** decides abnormality via the **reconstruction error**  $E$  or **Jensen-Shannon Divergence**  $JSD$  with a threshold  $t_i$  with a given classifier  $f$

$$E(x) = \|x - f_{\text{AE}}(x)\|_p > t_1 \text{ where } t_1 = \sup_{x \in X_{\text{val}}} \|x - f_{\text{AE}}(x)\|_p$$

where  $X_{\text{val}}$  is the validation data

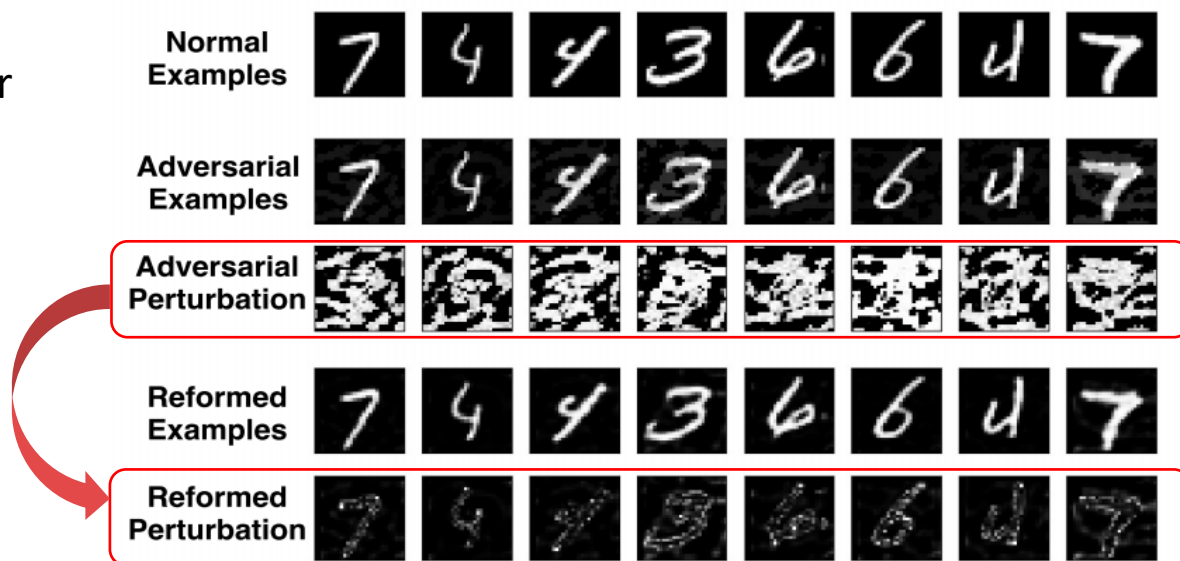
$$JSD(f_{\text{soft}}(x) || f_{\text{soft}}(f_{\text{AE}}(x))) > t_2 \text{ where } t_2 \text{ is a hyper-parameter}$$

where  $f_{\text{soft}}(x)$  is the softmax output of the classifier  $f$  on an input  $x$

- **Goal:** Training **Detector** and **Reformer** which are based on auto-encoders
- **Method:**
  - **Reformer:** Reform an input  $x$  to lie on the data manifold
    - Training an auto-encoder  $f_{\text{AE}}$  to minimize the loss over the training data  $X$

$$\ell(X) = \frac{1}{|X|} \sum_{x \in X} \|x - f_{\text{AE}}(x)\|_2$$

- To strengthen the network on adversarial attacks, **Detector** and **Reformer** are **randomly selected** from large number of trained **Detectors** and **Reformers** with different architectures
- **Experimental Results**
  - The Effect of Reformer



- **Experimental Results**

- MagNet [D. Meng et al., 2017] reports that it defends CW method

(a) MNIST

Attack	Norm	Parameter	No Defense	With Defense
Carlini	$L^2$		0.0%	99.5%
Carlini	$L^\infty$		0.0%	99.8%
Carlini	$L^0$		0.0%	92.0%

(b) CIFAR

Attack	Norm	Parameter	No Defense	With Defense
Carlini	$L^2$		0.0%	93.7%
Carlini	$L^\infty$		0.0%	83.0%
Carlini	$L^0$		0.0%	77.5%

- But, the results is broken [N. Carlini et al., 2017c]
  - CW method by performing 10,000 iterations of gradient descent

Dataset	Model	Success	Distortion ( $L_2$ )
MNIST	Unsecured	100%	1.64
	MagNet	99%	2.25
CIFAR	Unsecured	100%	0.30
	MagNet	100%	0.45

- **Motivation:** Pre-process an input to make a clean data via **WGAN** [M. Arjovsky et al., 2017]
  - MagNet [D. Meng et al., 2017] is based on auto-encoders
  - **WGAN:** For input data  $x$  and random vectors  $w$ , generator  $G$  and discriminator  $D$  minimize the following min-max loss  $V$

$$\min_G \max_D V(D, G) = \mathbb{E}_{p(x)} [D(x)] - \mathbb{E}_{p(w)} [D(G(w))]$$

- **Idea:** The **minimizer**  $\hat{w}$  of reconstruction error would generate the clean data

$$\text{Reconstruction error} = \|x - G(z)\|_2^2$$

- **Method:** Finding a minimizer  $\hat{w}$  of the reconstruction error for given generator  $G$  and input  $x$

$$\hat{w} = \underset{w}{\operatorname{argmin}} \|x - G(w)\|_2^2$$

- Choose initial  $w$  randomly
- Use Gradient Descent (GD) with some fixed step (e.g. 200-step)

\*min-max loss of original GAN:  $\min_G \max_D V(D, G) = \mathbb{E}_{p(x)} [\log D(x)] - \mathbb{E}_{p(z)} [\log(1 - D(G(z)))]$

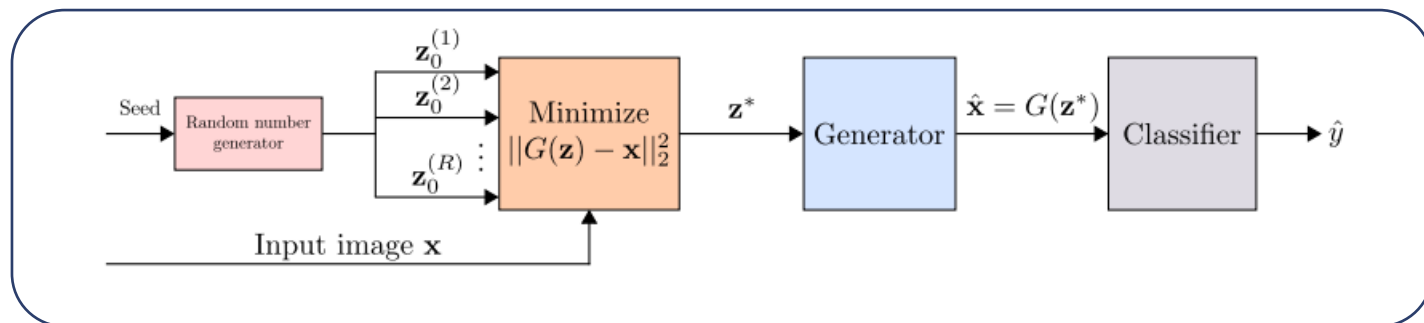
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- **Idea:** The **minimizer**  $\hat{w}$  of reconstruction error would generate the clean data

$$\text{Reconstruction error} = \|x - G(z)\|_2^2$$

- **Method:**
  - **Classifier** can be trained by using either **original** data or **reconstructed** data
  - The flow of **Defense-GAN** is

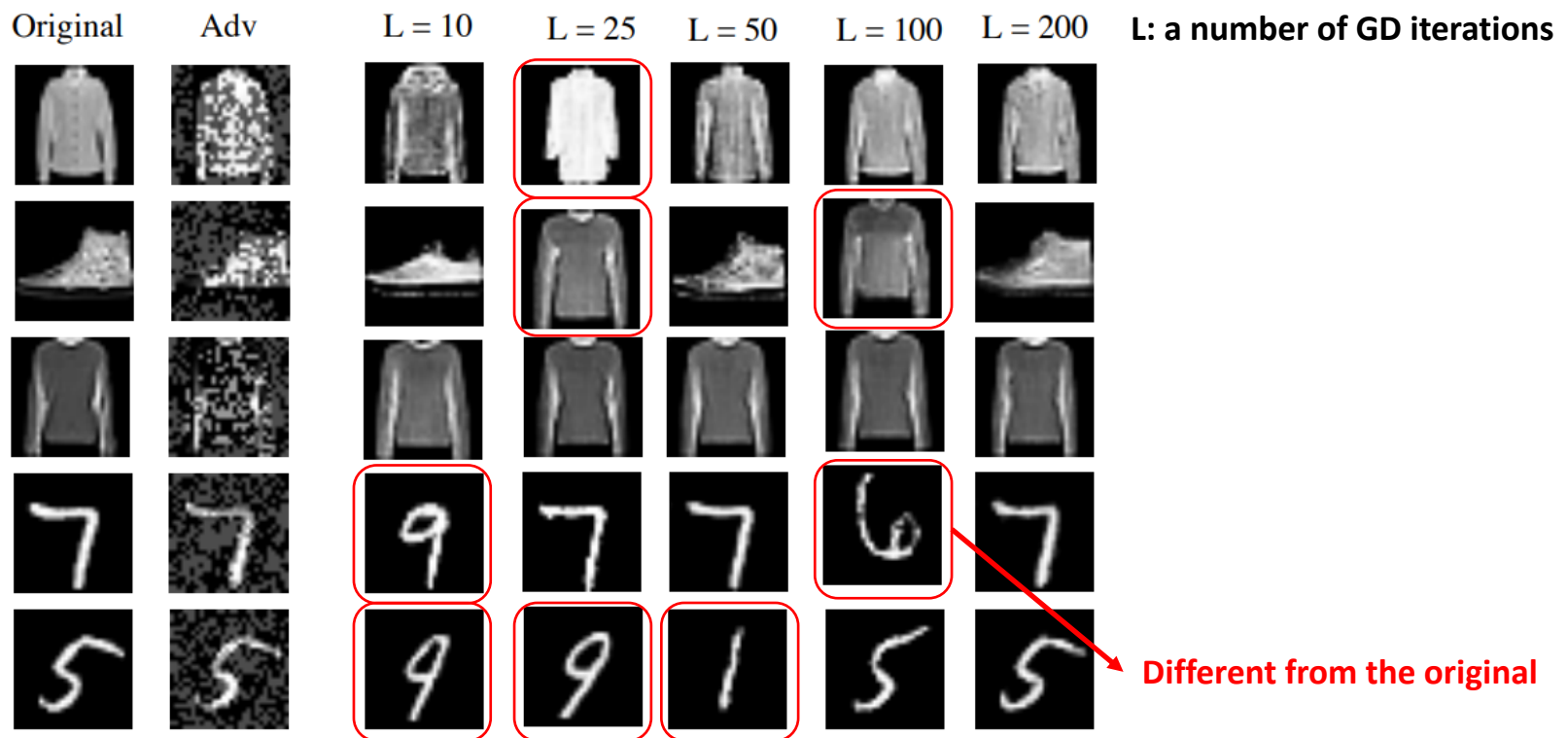




- **Experimental Result**

- The effect of **Generator**

- The reconstructed data are clean, but it often different from the **original**



Reconstructed data with various L

## • Experimental Result

- White-box attack with FGSM and CW method

	Attack	No Attack	No Defense	FGSM Adv. Training	Magnet	Defense-GAN
MNIST	FGSM					
	$\varepsilon = 0.3$	0.997	0.217	0.651	0.191	<b>0.988</b>
	CW					
	$l_2$ norm	0.997	0.141	0.077	0.038	<b>0.989</b>
	Attack	No Attack	No Defense	FGSM Adv. Training	Magnet	Defense-GAN
F-MNIST	FGSM					
	$\varepsilon = 0.3$	0.934	0.102	0.797	0.089	<b>0.879</b>
	CW					
	$l_2$ norm	0.934	0.076	0.157	0.060	<b>0.896</b>

- Black-box attack with FGSM

$\varepsilon$	MNIST	F-MNIST
0.10	0.9864	0.8844
0.15	0.9836	0.8267
0.20	0.9772	0.7492
0.25	0.9641	0.6384
0.30	0.9307	0.5126

### 1. Introduction

- What is adversarial example?
- Overview

### 2. Adversarial attack

- White-box attack
- Black-box attack

### 3. Adversarial Defense

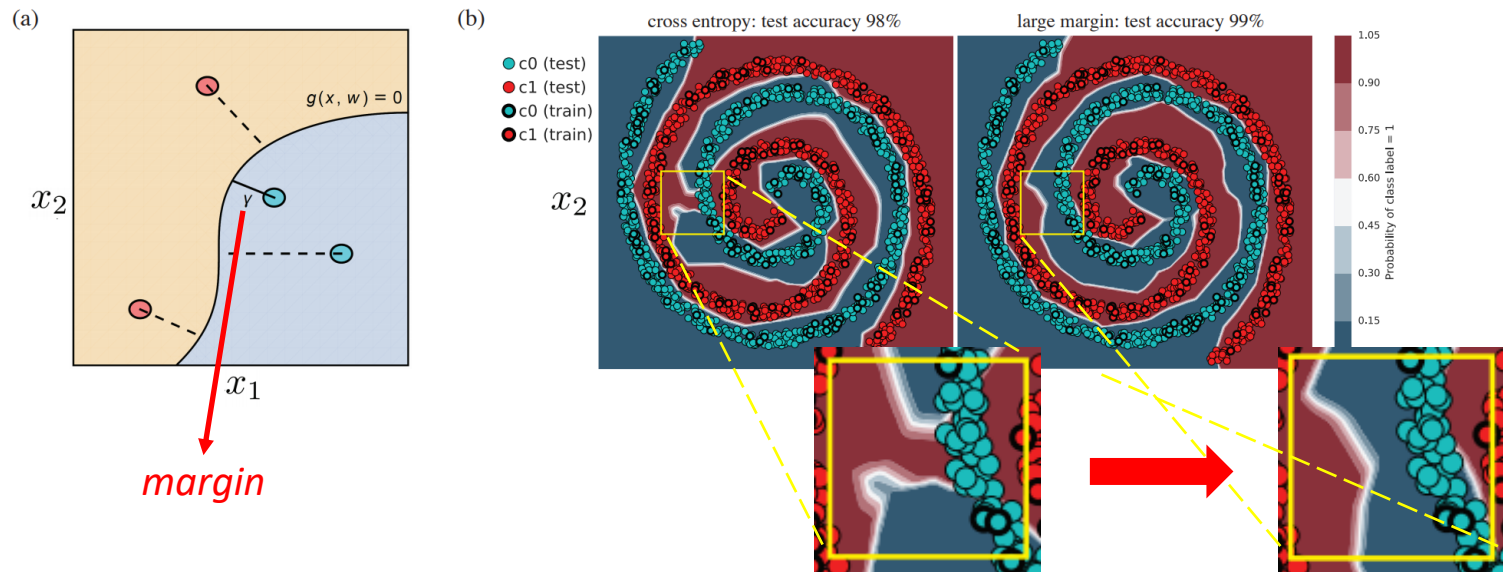
- Adversarial training
- Input pre-processing
- Robust network construction

- **Motivation:** Model robustness would be related to the margin of model

- **Idea:** Construct a model to have a large **margin**
  - Margin is the smallest distance from the training data to the decision boundary
  - Define a margin  $d$  as follow:

$$d = \min_{\delta} \|\delta\|_p \text{ s.t. } f_i(x + \delta) = f_j(x + \delta)$$

where  $f_i(\cdot)$  is  $i$ -th logit value of a classifier  $f(\cdot)$



- **Motivation:** Model robustness would be related to the **margin** of model

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where  $f_i(\cdot)$  is  $i$ -th logit value of a classifier  $f(\cdot)$

- Applying linear approximation to  $f_i$
- The approximated margin  $\hat{d}$  can be written in closed form as follow:

$$\hat{d} = \frac{|f_i(x) - f_j(x)|}{\|\nabla_x f_i(x) - \nabla_x f_j(x)\|_q}$$

where  $\|\cdot\|_q$  is the dual norm of  $\|\cdot\|_p$ ,  $q = \frac{p}{p-1}$

- **Goal:** Solving a new loss function to maximize the margin  $\hat{d}(\geq \gamma)$

$$\underset{\theta}{\text{minimize}} \sum_{(x,y) \sim \mathcal{D}} \mathcal{A}_{i \neq y} \max\left\{0, \gamma + \underbrace{\frac{f_i(x) - f_y(x)}{\|f_i(x) - f_y(x)\|}}_{\text{margin}}\right\}$$

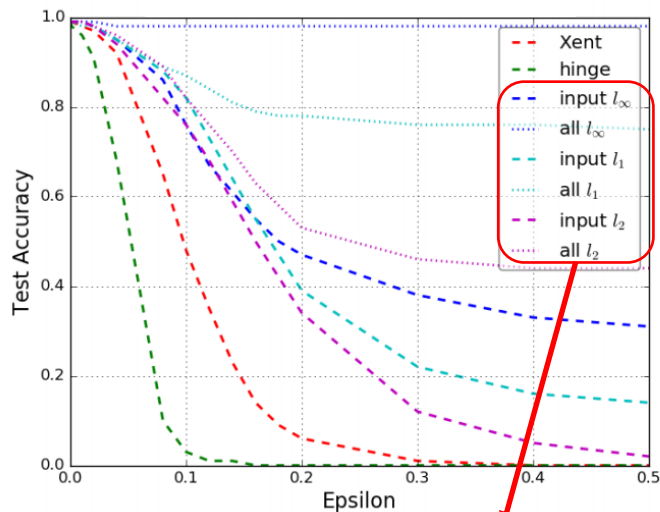
$\mathcal{A}$  : aggregate operator; max or sum  
 $\gamma$ : hyper-parameter

- **Method:** Adding cross-entropy loss with a small coefficient (less than 0.1%) to keep the classification performance
  - Various metrics can be used
    - $l_1, l_2, l_\infty$
  - The above margin is about input  $x$ , but it can be extended to hidden features

### • Experimental Result

- Test accuracy of standard model: 99.5%
- Test accuracy of the margin classifier models: 99.3~99.5%
- White-box: BIM attack
  - Xent: Cross-entropy loss

#### MNIST

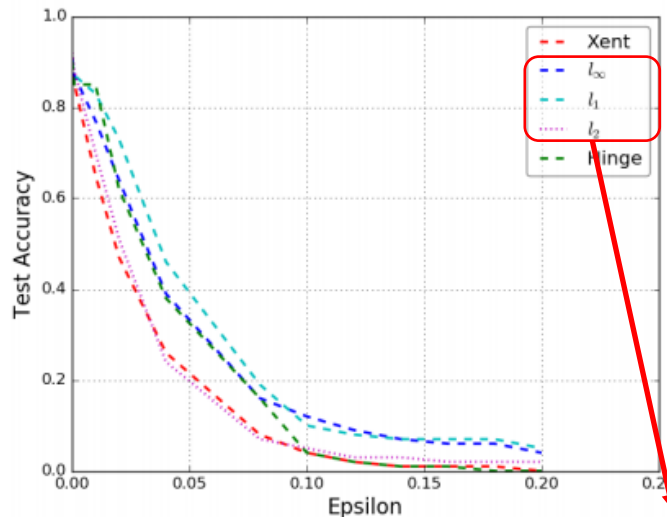


Large margin classifiers

Input  $l_p$ : applying the *margin loss* on first layer (input) only

All  $l_p$ : applying the *margin loss* on all hidden layer (hidden features)

#### CIFAR 10

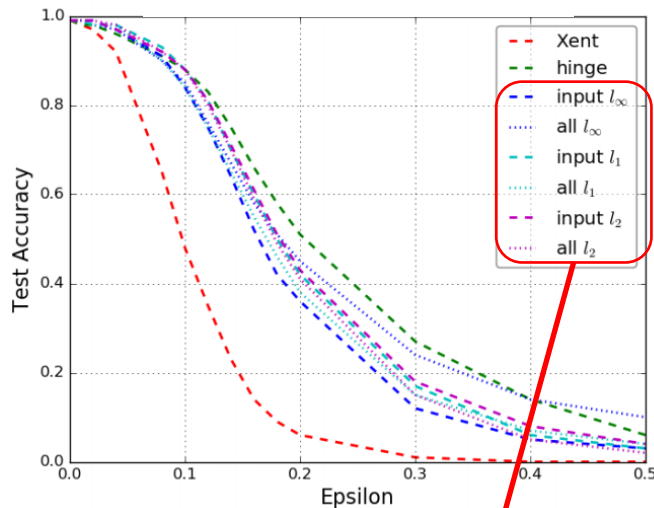


applying on multiple layer  
(input, output, some conv layers)

### • Experimental Result

- Test accuracy of standard model: 99.5%
- Test accuracy of the margin classifier models: 99.3~99.5%
- Black-box: BIM attack to Xent model
  - Xent: Cross-entropy loss

#### MNIST

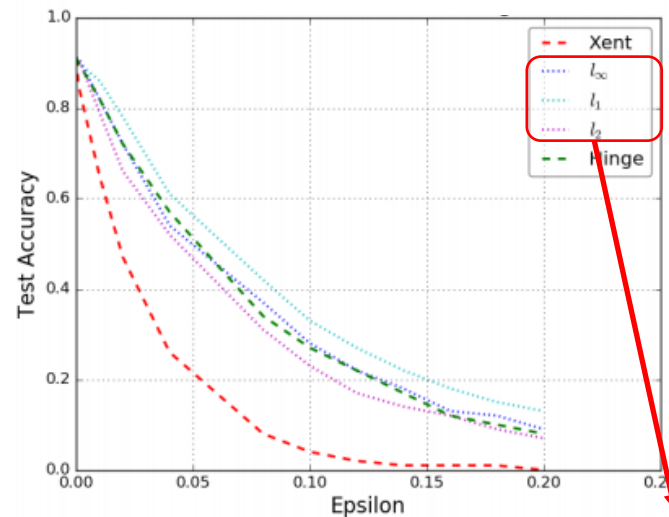


Large margin classifiers

Input  $l_p$ : applying the *margin loss* on first layer (input) only

All  $l_p$ : applying the *margin loss* on all hidden layer (hidden features)

#### CIFAR 10



applying on multiple layer  
(input, output, some conv layers)



- **Motivation:** Model robustness would be related to the **Lipchitz constant** of model
  - **Idea:** The global Lipchitz constant of model is bounded by the function of Lipchitz constants of all layers
  - **Goal:** Controlling the Lipchitz constants (**the spectral norm**) of all linear and convolutional layers to be smaller than 1
  - **Method:** Parseval regularization with  $R_k(\cdot)$ 
    - A weight matrix  $\mathcal{W}$  is called approximately a Parseval tight frame if  $\mathcal{W}$  satisfies  $\mathcal{W}^T \mathcal{W} \approx I$  where  $I$  is the identity matrix
    - Consider layer-wise regularizer  $R_k(\cdot)$  of a weight matrix as follow:

$$R_k(\mathcal{W}) = \frac{k}{2} \|\mathcal{W}^T \mathcal{W} - I\|_2^2$$

- Optimizing the regularizer  $R_k(\cdot)$  is expensive
  - The gradient of the regularizer  $R_k(\cdot)$  is follow:

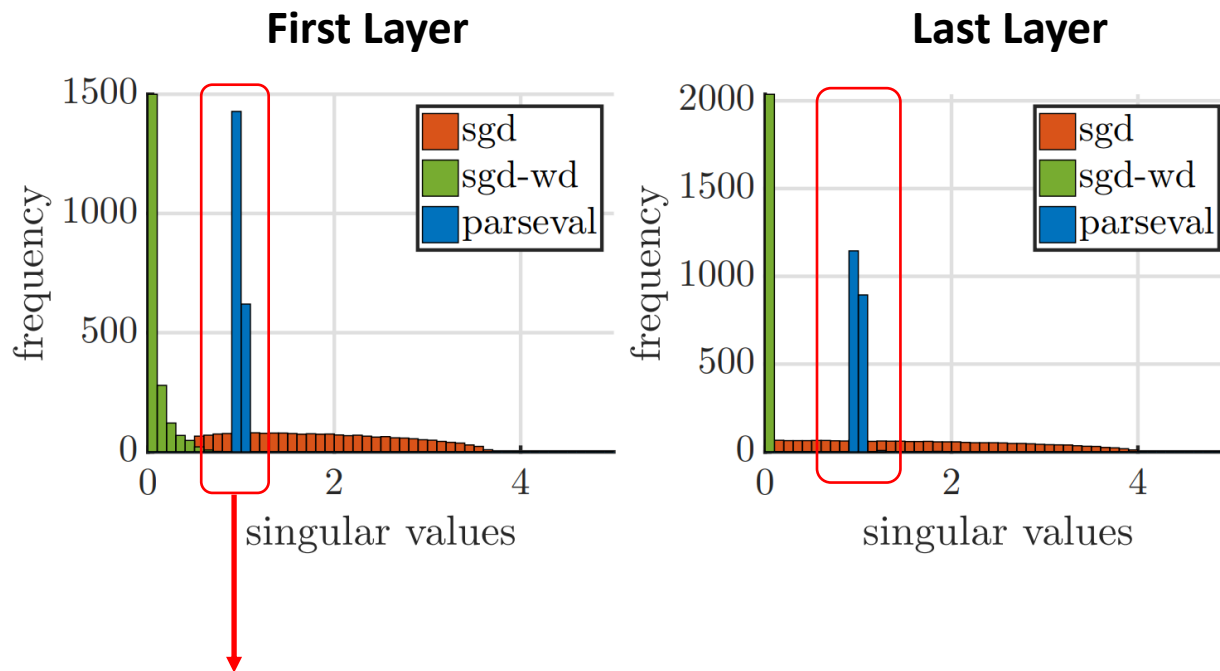
$$\nabla_{\mathcal{W}} R_k(\mathcal{W}) = k(\mathcal{W} \mathcal{W}^T - I) \mathcal{W}$$

- Performing the following update for regularizer to reduce the cost

$$\mathcal{W} \leftarrow (1 + k) \mathcal{W} - k \mathcal{W} \mathcal{W}^T \mathcal{W}$$

- **Experimental Result**

- The effect of the perseval regularization
  - Singular values of the weight matrices at the first and last layers of fully connected network in CIFAR10

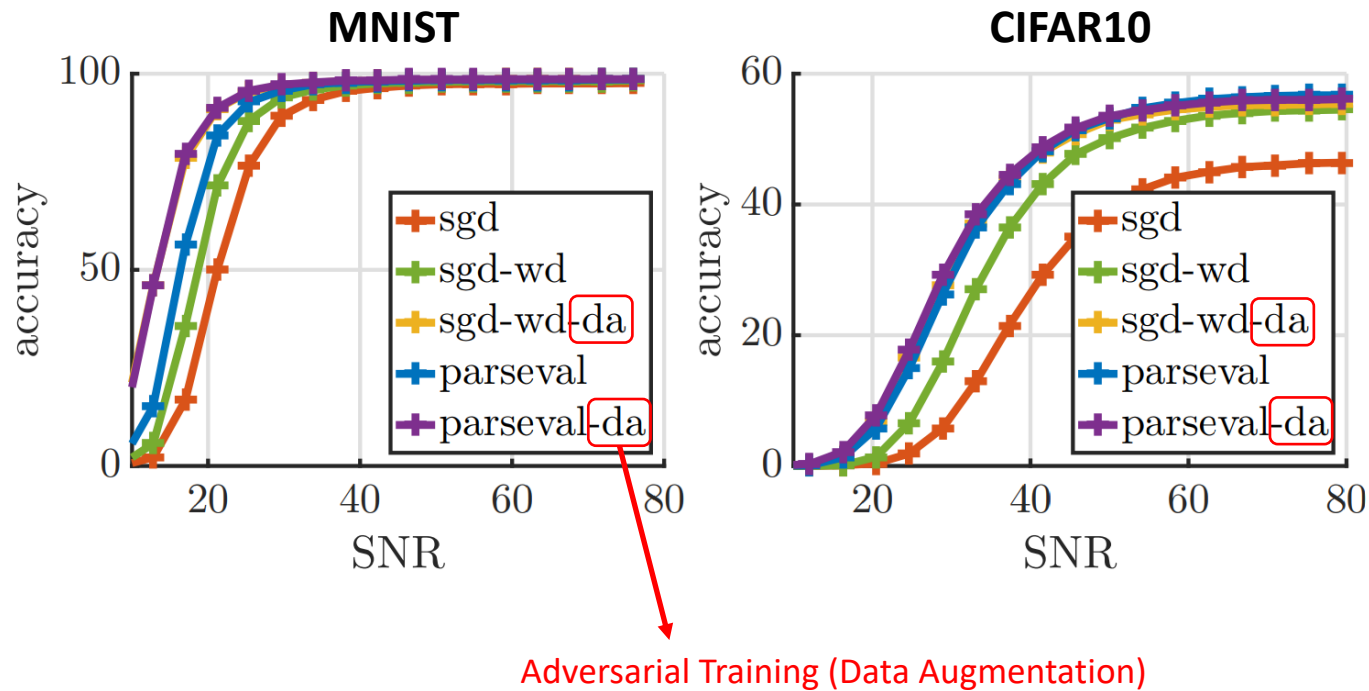


The largest singular value (the spectral norm) is almost 1

- **Experimental Result**

- White-box: FGSM attack
  - Signal to Noise Ratio (SNR)

$$\text{SNR}(x, \delta) = 20 \log \frac{\|x\|_2}{\|\delta\|_2}$$



- In this lecture, we cover threat of adversarial examples and various methods of adversarial attack and adversarial defense
  - Adversary could control the prediction of neural network via adversarial examples
- White-box attacks are mainly based on the gradient of model
  - FGSM, BIM, CW method
- Transferability of adversarial examples allow black-box attack
  - [Y. Liu et al., 2017]
  - [N. Papernot et al., 2017]
- There are many adversarial defense methods, but there is no perfect one
  - The most of defenses are heuristic

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