Meta Learning

Recent Advances in Deep Learning (Al602)
Lecture 15

Slide made by

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- Optimization-based meta-learning

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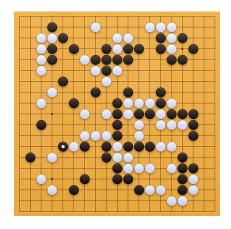
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- Overview of common approaches

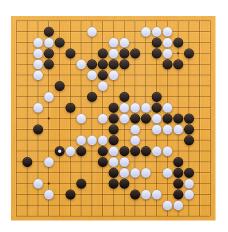
2. Approaches to Meta-learning

- Metric-based meta-learning
- Model-based meta-learning
- Optimization-based meta-learning

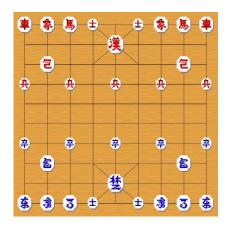
Learning: The model learns to solve a problem



• Meta-learning: The model learns to learn (fast adapt) new problems







New Boardgame?

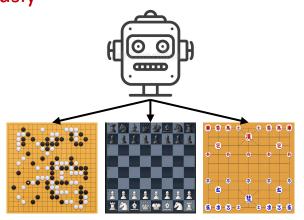
Multi-task learning vs Meta-learning

Multi-task learning:

• Given a pre-defined set of tasks $\{\mathcal{T}_1, \dots, \mathcal{T}_K\}$ (and corresponding loss functions $\{\mathcal{L}_i\}$), learn a single model f that solves all tasks simultaneously

Formally, the objective is given by

$$\underset{f}{\operatorname{argmin}} \sum_{k=1}^{K} \mathcal{L}_{k}(\mathcal{T}_{k}; \mathbf{f})$$



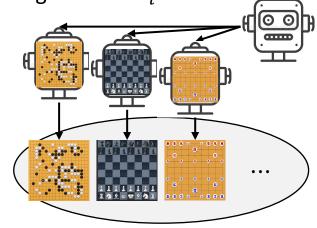
Meta-learning:

• For each task \mathcal{T}_i from a task distribution $p(\mathcal{T})$, learn a **meta-model** f that (quickly) learns a task-specific model $f_i \coloneqq f(\cdot | \mathcal{T}_i)$ that solves the given task \mathcal{T}_i

Formally, the objective is given by

$$\underset{f}{\operatorname{argmin}} \mathbb{E}_{\mathcal{T}_i} \mathcal{L}_i(\mathcal{T}_i; \underline{f_i})$$

Key difference: adaptation



Multi-task learning:

- Given a pre-defined set of tasks $\{\mathcal{T}_1, \dots, \mathcal{T}_K\}$ (and corresponding loss functions $\{\mathcal{L}_i\}$), learn a single model f that solves all tasks simultaneously
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Meta-learning:

- For each task \mathcal{T}_i from a task distribution $p(\mathcal{T})$, learn a **meta-model** f that (quickly) learns a task-specific model $f_i \coloneqq f(\cdot \mid \mathcal{T}_i)$ that solves the given task \mathcal{T}_i
- Formally, the objective is given by

$$\underset{f}{\operatorname{argmin}} \mathbb{E}_{\mathcal{T}_i} \mathcal{L}_i(\mathcal{T}_i; \underline{f_i})$$

• Since we mostly use parametric models (or deep neural network), we will denote the parameter of meta-model and task-specific models as θ and ϕ_i , respectively

Few-shot classification

- Human can classify novel objects even though they see only a few samples
- **Example:** Classify the breed of dogs (3-way 1-shot problem)







Q. What is the breed of this dog?

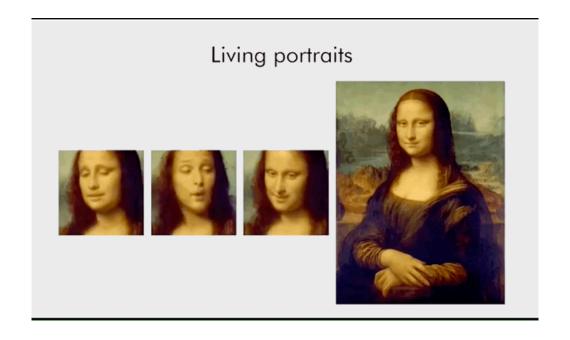


Few-shot classification

- Human can classify novel objects even though they see only a few samples
- Few-shot learning can be formulated as a meta-learning problem
 - **Task:** Given *N* classes of *K* samples each (i.e., *N*-way *K*-shot), predict the class of test samples (Each combination of *N* classes defines a task)
 - In this case, the meta model f learns a dog breed classifier $f_{\rm dog}$ from the given training images (and evaluated by test images)



- Few-shot classification
 - Classify novel instances with a few-shot of samples
- Few-shot generation
 - Generate novel instances of given samples
 - Example: Generate new emotions and angles of Mona Lisa (unique in the world!)



Few-shot classification

Classify novel instances with a few-shot of samples

Few-shot generation

Generate novel instances of given samples

Generalization of RL

Generalize to novel environments



Few-shot classification

Classify novel instances with a few-shot of samples

Few-shot generation

Generate novel instances of given samples

Generalization of RL

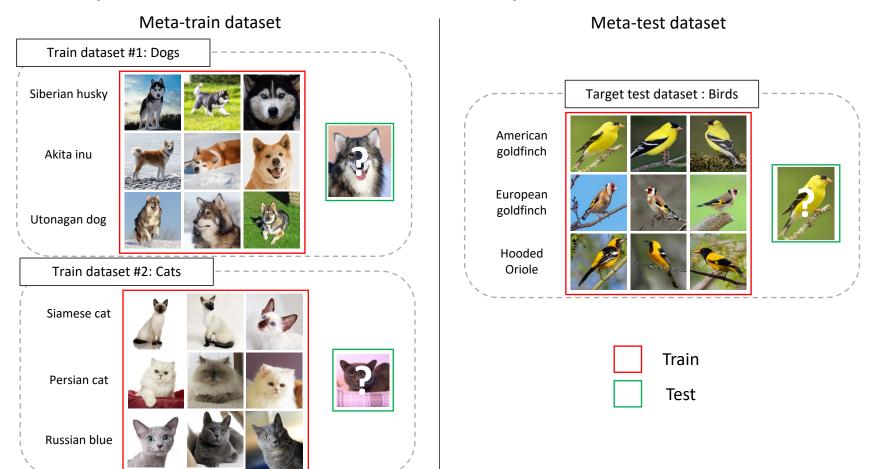
Generalize to novel environments

and LOTS of other applications

- Neural architecture search
- Hyperparameter optimization
- Loss function design
- ...and so on

Problem formulation

- To meta-learn a model, we need a meta-train dataset $\{(\mathcal{D}_i^{\text{train}}, \mathcal{D}_i^{\text{test}})\}$ consist of training and test datasets for each task \mathcal{T}_i
- The performance of meta model is evaluated by a meta-test dataset



General recipe for meta-learning

- The core of meta-learning is how to learn a task-specific models for a given task
- There are two common ways to *learn* the model from the dataset $\mathcal{D}_i^{ ext{train}}$

Model-based meta-learning

- The meta-parameter heta is fixed, and the task is encoded to a context variable c_i
- Namely, the task-specific function is given by $f(\cdot | \theta, c_i)$

Optimization-based meta-learning

- Learn a parameter $\phi_i = g(\mathcal{D}_i^{\text{train}}; \theta)$ for each task \mathcal{T}_i
- Namely, the task-specific function is given by $f(\cdot|\phi_i)$
- Note that deep learning procedure can be decomposed into two steps:
 - How to set the initial parameter $\phi_i^{(0)}$
 - How to update the parameter $\phi_i^{(t)}$ to the better parameter $\phi_i^{(t+1)}$
- The meta-learner θ will learn the initialization and/or update schemes

Metric-based meta-learning

- For a special type of meta-learning, few-shot classification, another common approach is to learn an embedding function and the corresponding metric
- The embedding function maps similar samples to the similar embedding, and one can classify a novel sample by finding the nearest cluster

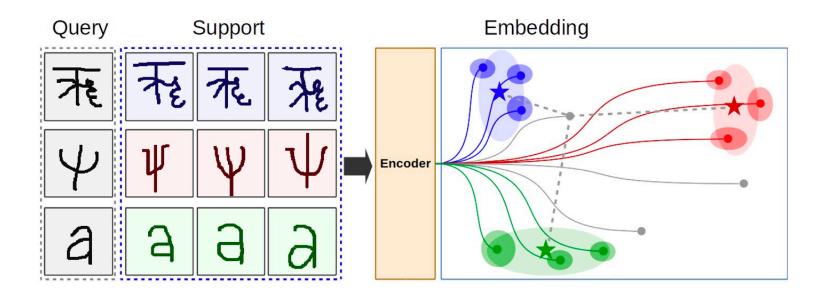


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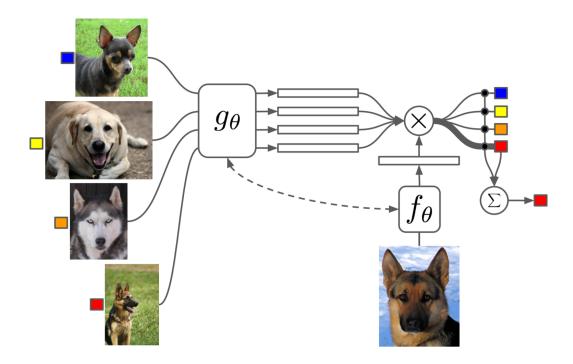
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Matching Networks [Vinyals et al. 16] propose to learn a shared embedding space over multiple subclassification problems.



Matching network training objective:

$$\theta = \arg \max_{\theta} E_{L \sim T} \left[E_{S \sim L, B \sim L} \left[\sum_{(x,y) \in B} \log P_{\theta} (y|x, S) \right] \right]$$

Obtaining the optimal θ can be done via **episodic training**.

- First sample L (label set) from T, and use L to sample the support set S and a batch B.
- Then minimize the error predicting the labels in the batch B conditioned on the support set S.

Matching Networks

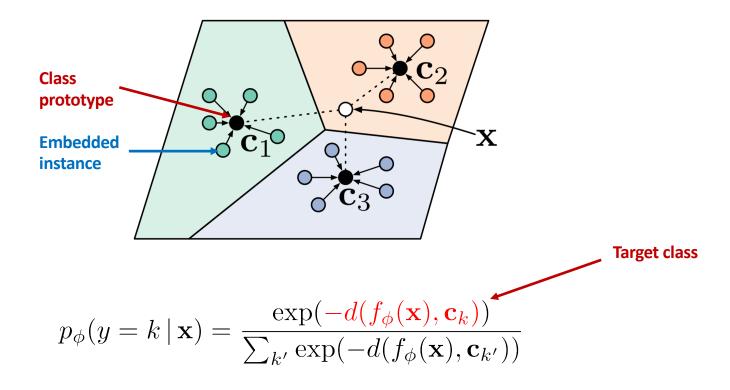
 Matching Networks generalize well and thus outperforms baseline classifiers and meta-learning models (MANN) on few-shot learning tasks.

Model	Matching Fn	Fine Tune	5-w 1-shot	ay Acc 5-shot	20-w 1-shot	ay Acc 5-shot
PIXELS	Cosine	N	41.7%	63.2%	26.7%	42.6%
BASELINE CLASSIFIER	Cosine	N	80.0%	95.0%	69.5%	89.1%
BASELINE CLASSIFIER	Cosine	Y	82.3%	98.4%	70.6%	92.0%
BASELINE CLASSIFIER	Softmax	Y	86.0%	97.6%	72.9%	92.3%
MANN (No Conv) [21]	Cosine	N	82.8%	94.9%	_	_
CONVOLUTIONAL SIAMESE NET [11]	Cosine	N	96.7%	98.4%	88.0%	96.5%
CONVOLUTIONAL SIAMESE NET [11]	Cosine	Y	97.3%	98.4%	88.1%	97.0%
MATCHING NETS (OURS)	Cosine	N	98.1%	98.9%	93.8%	98.5%
MATCHING NETS (OURS)	Cosine	Y	97.9%	98.7%	93.5%	98.7 %

Table 1: Results on the Omniglot dataset.

 Fine-tuning helped with baseline classifiers, but not in the case of Matching Networks.

Prototypical Networks [Snell et al. 17] use meta-learning to learn a metric space that minimizes the Euclidean distance between the prototypes and each training instance.



Prototypical Networks are trained by minimizing the negative log-probability

$$J(\phi) = -\log p_{\phi}(y = k \mid \mathbf{x})$$
 via episodic training.

```
Input: Training set \mathcal{D} = \{(\mathbf{x}_1, y_1), \dots, (\mathbf{x}_N, y_N)\}, where each y_i \in \{1, \dots, K\}. \mathcal{D}_k denotes the
   subset of \mathcal{D} containing all elements (\mathbf{x}_i, y_i) such that y_i = k.
Output: The loss J for a randomly generated training episode.
   V \leftarrow \text{RANDOMSAMPLE}(\{1, \dots, K\}, N_C)
                                                                                    for k in \{1, ..., N_C\} do
      S_k \leftarrow \text{RANDOMSAMPLE}(\mathcal{D}_{V_k}, N_S)
                                                                                            Q_k \leftarrow \mathsf{RANDOMSAMPLE}(\mathcal{D}_{V_k} \setminus S_k, N_Q)

    Select query examples

     \mathbf{c}_k \leftarrow \frac{1}{N_C} \sum_{i} f_{\phi}(\mathbf{x}_i)
                                                                    end for
   J \leftarrow 0
                                                                                                          ▶ Initialize loss
   for k in \{1, ..., N_C\} do
      for (\mathbf{x}, y) in Q_k do
         J \leftarrow J + \frac{1}{N_C N_O} \left[ d(f_{\phi}(\mathbf{x}), \mathbf{c}_k)) + \log \sum_{i,j} \exp(-d(f_{\phi}(\mathbf{x}), \mathbf{c}_{k'})) \right]

    □ Update loss

      end for
   end for
```

Prototypical Networks

 Prototypical Networks outperform Matching Networks and MAML on few-shot classification tasks.

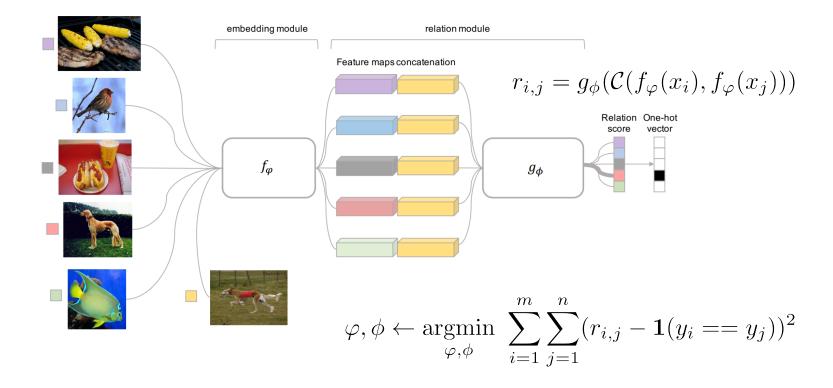
Omniglot

			5-way Acc.		20-way Acc.	
Model	Dist.	Fine Tune	1-shot	5-shot	1-shot	5-shot
MATCHING NETWORKS [32]	Cosine	N	98.1%	98.9%	93.8%	98.5%
MATCHING NETWORKS [32]	Cosine	Y	97.9%	98.7%	93.5%	98.7%
NEURAL STATISTICIAN [7]	-	N	98.1%	99.5%	93.2%	98.1%
MAML [9]*	-	N	98.7%	99.9%	95.8%	98.9%
PROTOTYPICAL NETWORKS (OURS)	Euclid.	N	98.8%	99.7%	96.0%	98.9%

minilmageNet

			5-way Acc.			
Model	Dist.	Fine Tune	1-shot	5-shot		
BASELINE NEAREST NEIGHBORS*	Cosine	N	$28.86 \pm 0.54\%$	$49.79 \pm 0.79\%$		
MATCHING NETWORKS [32]*	Cosine	N	$43.40 \pm 0.78\%$	$51.09 \pm 0.71\%$		
MATCHING NETWORKS FCE [32]*	Cosine	N	$43.56 \pm 0.84\%$	$55.31 \pm 0.73\%$		
META-LEARNER LSTM [24]*	-	N	$43.44 \pm 0.77\%$	$60.60 \pm 0.71\%$		
MAML [9]	-	N	$\textbf{48.70} \pm \textbf{1.84\%}$	$63.15 \pm 0.91\%$		
PROTOTYPICAL NETWORKS (OURS)	Euclid.	N	$\textbf{49.42} \pm \textbf{0.78}\%$	$\textbf{68.20} \pm \textbf{0.66}\%$		

Relation Networks [Sung et al. 18] learns to learn a deep metric space by learning to minimize the relation scores between the query and the support samples.



Relation Networks outperforms Prototypical Networks and MAML on few-shot learning tasks.

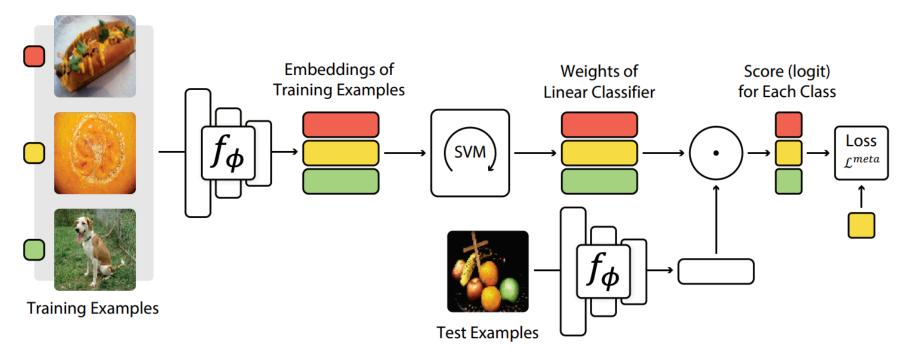
Omniglot

Model	Fine Tune	5-way	y Acc.	20-way Acc.		
		1-shot	5-shot	1-shot	5-shot	
MANN [32]	N	82.8%	94.9%	-	-	
CONVOLUTIONAL SIAMESE NETS [20]	N	96.7%	98.4%	88.0%	96.5%	
CONVOLUTIONAL SIAMESE NETS [20]	Y	97.3%	98.4%	88.1%	97.0%	
MATCHING NETS [39]	N	98.1%	98.9%	93.8%	98.5%	
MATCHING NETS [39]	Y	97.9%	98.7%	93.5%	98.7%	
SIAMESE NETS WITH MEMORY [18]	N	98.4%	99.6%	95.0%	98.6%	
NEURAL STATISTICIAN [8]	N	98.1%	99.5%	93.2%	98.1%	
META NETS [27]	N	99.0%	-	97.0%	-	
PROTOTYPICAL NETS [36]	N	98.8%	99.7%	96.0%	98.9%	
MAML [10]	Y	$98.7 \pm 0.4\%$	$\textbf{99.9} \pm \textbf{0.1}\%$	$95.8 \pm 0.3\%$	$98.9 \pm 0.2\%$	
RELATION NET	N	$99.6 \pm 0.2\%$	99.8± 0.1%	$97.6 \pm 0.2\%$	99.1± 0.1%	

miniImageNet

Model	FT	5-way	Acc.
		1-shot	5-shot
MATCHING NETS [39]	N	$43.56 \pm 0.84\%$	$55.31 \pm 0.73\%$
META NETS [27]	N	$49.21 \pm 0.96\%$	-
META-LEARN LSTM [29]	N	$43.44 \pm 0.77\%$	$60.60 \pm 0.71\%$
Maml [10]	Y	$48.70 \pm 1.84\%$	$63.11 \pm 0.92\%$
PROTOTYPICAL NETS [36]	N	$49.42 \pm 0.78\%$	$68.20 \pm 0.66\%$
RELATION NET	N	$50.44 \pm 0.82\%$	$65.32 \pm 0.70\%$

 MetaOptNet [Lee et al. 19] uses more complex classifiers (e.g., SVM) instead of the naïve nearest neighbor classifier, upon the learned embedding



 Here, the classifier is defined by a closed form solution of some quadratic programming (QP) problem

 $heta = \mathcal{A}(\mathcal{D}^{train}; oldsymbol{\phi}) = rg \min_{\{oldsymbol{w}_k\}} \min_{\{\xi_i\}} rac{1}{2} \sum_k ||oldsymbol{w}_k||_2^2 + C \sum_n \xi_n$

 MetaOptNet with ridge regression (RR) and support vector machine (SVM) shows better results than naïve prototypical network

miniImageNet 5-way				tieredImageNet 5-way				
	1-s	shot	5-s	hot	1-8	shot	5-s	hot
model	acc. (%)	time (ms)	acc. (%)	time (ms)	acc. (%)	time (ms)	acc. (%)	time (ms)
4-layer conv (feature dimension=	:1600)							
Prototypical Networks [17, 28]	$53.47_{\pm0.63}$	$6_{\pm0.01}$	$70.68_{\pm 0.49}$	$7_{\pm0.02}$	$54.28_{\pm 0.67}$	$6_{\pm0.03}$	$71.42_{\pm 0.61}$	$7_{\pm0.02}$
MetaOptNet-RR (ours)	$53.23_{\pm 0.59}$	$20_{\pm0.03}$	$69.51_{\pm 0.48}$	$27_{\pm0.05}$	$54.63_{\pm 0.67}$	$21_{\pm0.05}$	$72.11_{\pm 0.59}$	$28_{\pm0.06}$
MetaOptNet-SVM (ours)	$52.87_{\pm 0.57}$	$28_{\pm0.02}$	$68.76 \scriptstyle{\pm 0.48}$	$37_{\pm0.05}$	$54.71_{\pm 0.67}$	$28_{\pm0.07}$	$71.79_{\pm 0.59}$	$38_{\pm0.08}$
ResNet-12 (feature dimension=1	6000)							
Prototypical Networks [17, 28]	$59.25_{\pm 0.64}$	$60_{\pm 17}$	$75.60_{\pm 0.48}$	$66_{\pm 17}$	$61.74_{\pm 0.77}$	$61_{\pm 17}$	$80.00_{\pm 0.55}$	$66_{\pm 18}$
MetaOptNet-RR (ours)	$61.41_{\pm 0.61}$	$68_{\pm 17}$	77.88 _{±0.46}	$75_{\pm 17}$	65.36 _{±0.71}	$69_{\pm 17}$	81.34 _{±0.52}	$77_{\pm 17}$
MetaOptNet-SVM (ours)	62.64 $_{\pm 0.61}$	$78_{\pm 17}$	78.63 $_{\pm 0.46}$	$89_{\pm 17}$	65.99 _{±0.72}	$78_{\pm 17}$	81.56 _{±0.53}	$90_{\pm 17}$

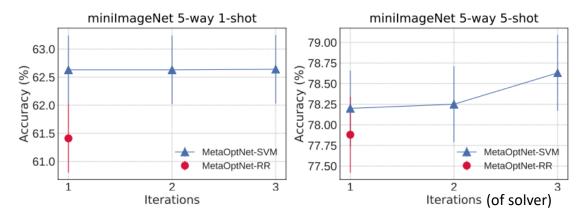


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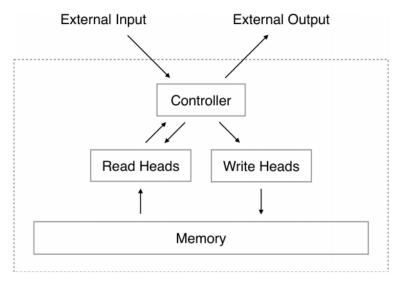
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- Overview of common approaches

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- Optimization-based meta-learning

Meta-Learning with MANN

- [Graves et al. 14] propose a Neural Turing Machine (NTM), a neural networks architecture which has external memory.
- With an explicit storage buffer, it is easier for the network to rapidly incorporate new information and not to forget in the future.
- [Santoro et al. 16] proposed memory-augmented neural network (MANN) to rapidly assimilate new data, and to make accurate predictions with few samples.

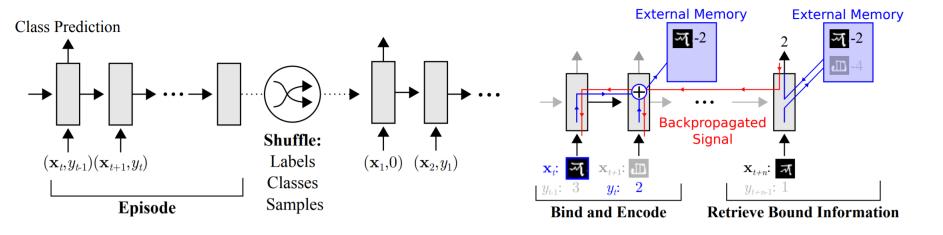


Neural Turing Machine

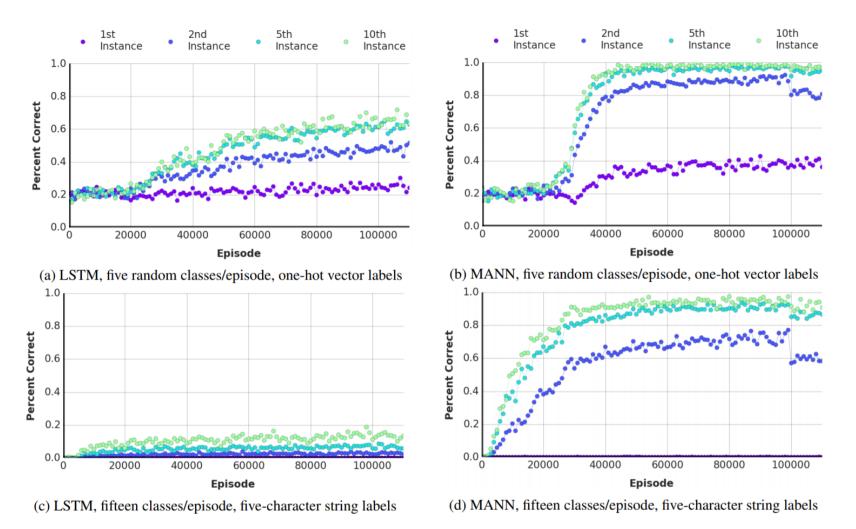
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Meta-Learning with MANN

- They train MANN to perform classification while presenting the data instance and labels in a time-offset manner to prevent simple mapping from label to label.
- Further, they shuffle labels, classes, and samples from time to time to prevent weights from binding to sample-class binding.



 This method enables to learn a generic scheme to bind representations to their appropriate labels regardless of the actual contents of data representations or labels. MANN significantly outperforms LSTM (which has internal memory) for fewshot classification on Omniglot dataset.

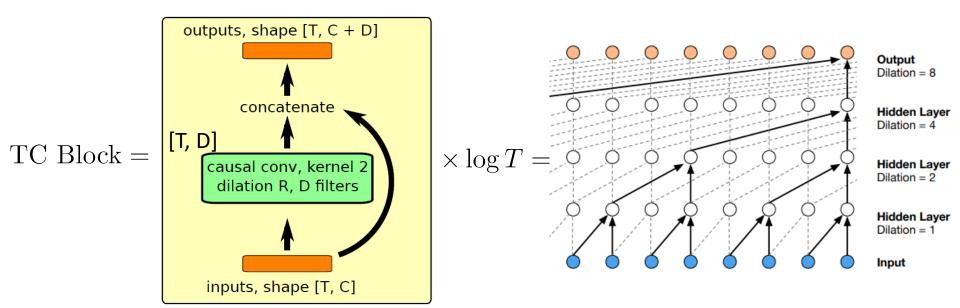


- Traditional RNN architectures propagate information by keeping it in their hidden state from one time step to the next.
 - This temporally-linear dependency bottlenecks their capacity.
- [Mishra et al. 18] propose a model architectures that addresses this shortcoming.
- They combine these two modules for simple neural attentive learner (SNAIL):
 - Temporal convolutions, which enable the meta-learner to aggregate contextual information from past experience
 - Causal attention, which allow it to pinpoint specific pieces of information within that context.
- These two components complement each other: while the former provide highbandwidth access at the expense of finite context size, the latter provide pinpoint access over an infinitely large context.

- Two of the building blocks that compose SNAIL architectures.
- A Dense block applies a causal 1D-convolution, and then concatenates the output to its input. A Temporal Convolution (TC) block applies a series of dense blocks with exponentially-increasing dilation rates.

```
1: function TCBLOCK(inputs, sequence length T, number of filters D):
```

- for i in $1, \ldots, \lceil \log_2 T \rceil$ do
- inputs = DenseBlock(inputs, 2^i , D) 3:
- return inputs 4:

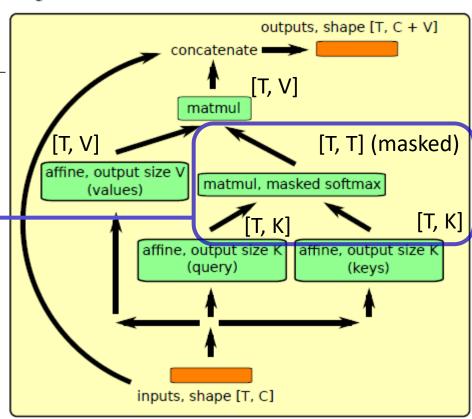


Dense Block

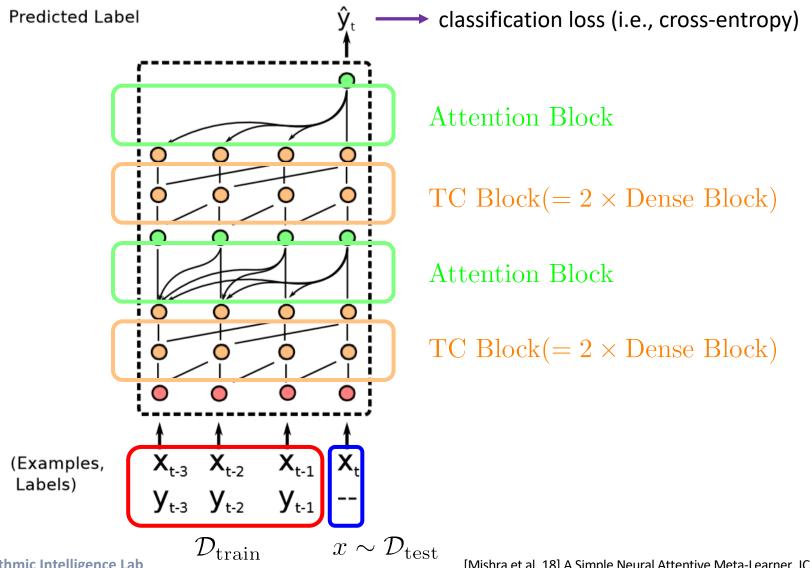
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- Two of the building blocks that compose SNAIL architectures.
- A attention block performs a causal key-value lookup and also concatenates the output to the input; they style this operation after the self-attention mechanism.
 - 1: **function** ATTENTIONBLOCK(inputs, key size K, value size V):
 - 2: keys, query = affine(inputs, K), affine(inputs, K)
 - 3: logits = matmul(query, transpose(keys))
 - 4: probs = CausallyMaskedSoftmax(logits / \sqrt{K})
 - 5: values = affine(inputs, V)
 - 6: read = matmul(probs, values)
 - 7: **return** concat(inputs, read)

Self-attention relates different positions of a single sequence in order to compute a representation



Overview of the SNAIL for supervised learning:

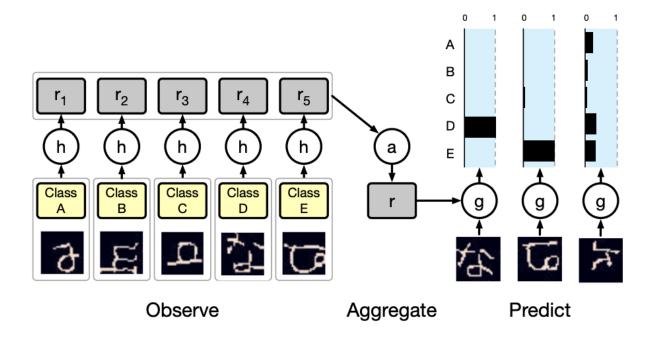


- SNAIL outperforms state-of-the-art methods in few-shot classification tasks that are extensively hand-designed, and/or domain-specific (e.g., Matching networks [Vinyals et al. 16]).
- It significantly exceeds the performance of methods such as MANN that are similarly simple and generic.

Method	5-Way (Omniglot	20-Way Omniglot		
	1-shot	5-shot	1-shot	5-shot	
Santoro et al. (2016)	82.8%	94.9%	_	-	
Koch (2015)	97.3%	98.4%	88.2%	97.0%	
Vinyals et al. (2016)	98.1%	98.9%	93.8%	98.5%	
Finn et al. (2017)	98.7% \pm 0.4%	99.9% \pm 0.3%	$95.8\% \pm 0.3\%$	$98.9\% \pm 0.2\%$	
Snell et al. (2017)	97.4%	99.3%	96.0%	98.9%	
Munkhdalai & Yu (2017)	98.9%	_	97.0%	_	
SNAIL, Ours	\parallel 99.07% \pm 0.16%	99.78% ± 0.09%	97.64% ± 0.30%	99.36% \pm 0.18%	

Method	5-Way Min	i-ImageNet
	1-shot	5-shot
Vinyals et al. (2016)	43.6%	55.3%
Finn et al. (2017)	$48.7\% \pm 1.84\%$	$63.1\% \pm 0.92\%$
Ravi & Larochelle (2017)	$43.4\% \pm 0.77\%$	$60.2\% \pm 0.71\%$
Snell et al. (2017)	$46.61\% \pm 0.78\%$	$65.77\% \pm 0.70\%$
Munkhdalai & Yu (2017)	$49.21\% \pm 0.96\%$	_
SNAIL, Ours	$ $ 55.71% \pm 0.99%	$ $ 68.88% \pm 0.92%

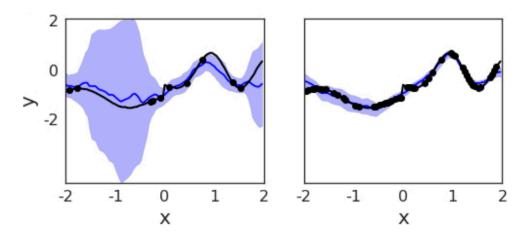
Conditional Neural Process (CNP) [Garnelo et al. 18] extracts the context variable of task with set encoder, and predicts target under the context



• Given observation O_N , model predicts outputs for both observed and unobserved samples, and trained to maximize the likelihood

$$\mathcal{L}(\theta) = -\mathbb{E}_{f \sim P} \left[\mathbb{E}_{N} \left[\log Q_{\theta}(\{y_{i}\}_{i=0}^{n-1} | O_{N}, \{x_{i}\}_{i=0}^{n-1}) \right] \right]$$

 Conditional Neural Process (CNP) behaves like a neural version of Gaussian process, e.g., it can predict uncertainty of outputs



 CNP is also computationally efficient as the input information is amortized to a single context variable, hence it has linear complexity

	5-way Acc		20-way Acc		Runtime	Omniglot
	1-shot	5-shot	1-shot	5-shot		classification
MANN	82.8%	94.9%	-	-	$\mathcal{O}(nm)$	
MN	98.1%	98.9%	93.8%	98.5%	$\mathcal{O}(nm)$	
CNP	95.3%	98.5%	89.9%	96.8%	$\mathcal{O}(n+m)$	

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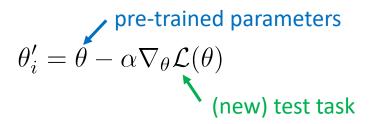
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 - Learning model initialization
 - Learning optimizers

Learning Good Initialization for Few-Shot Learning

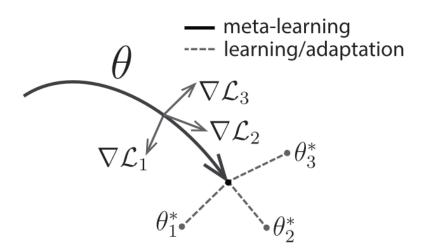
- Few-shot learning tackles limited-data scenario
 - One way to overcome the lack of data is initialization
- Common initialization method: pre-train with ImageNet and fine-tune
 - (+) Generally works very well on various tasks
 - (-) **Not work** when one has **only** a small number of examples (1-shot, 5-shot, etc.)
 - (-) Cannot be used when target network architectures are different from source model



- Learning initializations of a network that
 - Adapt fast with a small number of examples (few-shot learning)
 - Simple and easily generalized to various model architecture and tasks

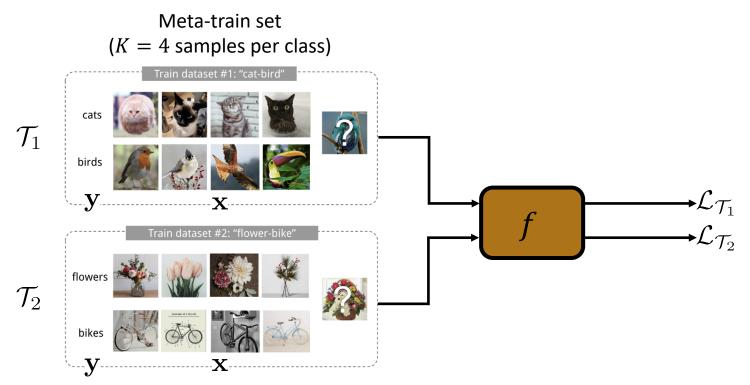
Model-Agnostic Meta-Learning (MAML)

- Key idea
 - Train over many tasks, to learn parameter θ that transfers well
 - Use objective that **encourage** heta to **fast adapt** when fine-tuned with small data
 - Assumption: some representations are more transferrable than others
- Model find parameter heta that would reduce the validation loss on each task
 - To do that, find (one or more steps of) fine-tuned parameter from θ for each task
 - And reduce the validation loss at fine-tuned parameter for each task
 - Meta-update the θ to direction that would adapt faster on each new task



Model-Agnostic Meta-Learning (MAML)

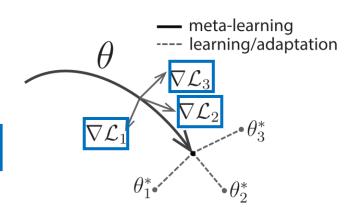
- Notations and problem set-up
 - Task $\mathcal{T} = \{\mathbf{x}, \mathbf{y}, \mathcal{L}(\mathbf{x}, \mathbf{y})\}$
 - Consider a distribution over tasks $p(\mathcal{T})$
 - Model is trained to learn new task $\mathcal{T}_i \sim p(\mathcal{T})$ from only K samples
 - Loss function for task \mathcal{T}_i is $\mathcal{L}_{\mathcal{T}_i}$
 - Model f is learned by minimizing the test error on new samples from \mathcal{T}_i



Algorithms

- Consider a model $f_{ heta}$ parameterized with heta
- Inner-loop
 - Adapting model to a new task \mathcal{T}_i

$$\theta_i' = \theta - \alpha \nabla_{\theta} \mathcal{L}_{\mathcal{T}_i}(f_{\theta})$$



Where α is learning rate,

- We can compute θ'_i with one or more gradient descent update steps
- Outer-loop
 - Model parameters are trained by optimizing the performance of $f_{ heta_i'}$
 - With respect to θ across tasks sampled from $p(\mathcal{T})$

$$\min_{\theta} \sum_{\mathcal{T}_i \sim p(\mathcal{T})} \mathcal{L}_{\mathcal{T}_i}(f_{\theta_i'}) = \sum_{\mathcal{T}_i \sim p(\mathcal{T})} \mathcal{L}_{\mathcal{T}_i}\left(f_{\theta - \alpha \nabla_{\theta} \mathcal{L}_{\mathcal{T}_i}(f_{\theta})}\right)$$

• So, the meta-optimization:

$$\theta \leftarrow \theta - \beta \nabla_{\theta} \sum_{\mathcal{T}_i \sim p(\mathcal{T})} \mathcal{L}_{\mathcal{T}_i}(f_{\theta_i'})$$

Where β is meta-learning rate

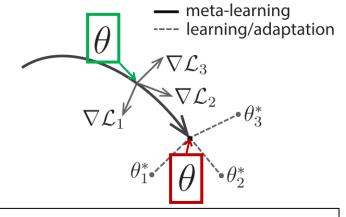
Algorithms

- Consider a model $f_{ heta}$ parameterized with heta
- Inner-loop
 - Adapting model to a new task \mathcal{T}_i

$$\theta_i' = \theta - \alpha \nabla_{\theta} \mathcal{L}_{\mathcal{T}_i}(f_{\theta})$$

Where α is learning rate,





 θ that would adapt better than θ

- Outer-loop
 - Model parameters are trained by optimizing the performance of $f_{ heta_i'}$

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Where β is meta-learning rate

Meta-Gradients of MAML

- MAML computes 2nd gradients
 - 1-step optimization example

Task-specificly optimized parameters

Meta-learned initial model parameters

$$g_{\text{MAML}} = \nabla_{\theta} \mathcal{L}_{\mathcal{T}_{i}}(f_{\theta})$$

$$g_{\text{MAML}} = \nabla_{\theta} \mathcal{L}_{\mathcal{T}_{i}}(\theta') = (\nabla_{\theta'} \mathcal{L}_{\mathcal{T}_{i}}(f_{\theta'})) \cdot (\nabla_{\theta} \theta')$$

$$= (\nabla_{\theta'} \mathcal{L}_{\mathcal{T}_{i}}(f_{\theta'})) \cdot (\nabla_{\theta}(\theta - \alpha \nabla_{\theta} \mathcal{L}_{\mathcal{T}_{i}}(f_{\theta})))$$

- High computation cost
- Computation cost is increased with a number of inner-loop iterations T

First Order Approximation of MAML

- MAML computes 2nd gradients
 - 1-step optimization example

Task-specificly optimized parameters

Meta-learned initial model parameters

$$g_{\text{MAML}} = \nabla_{\theta} \mathcal{L}_{\mathcal{T}_{i}}(f_{\theta})$$

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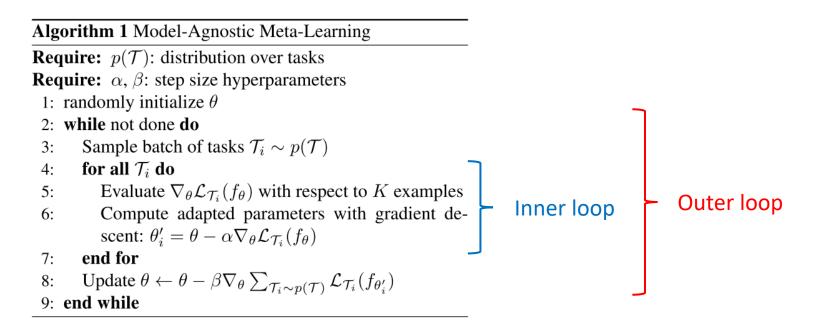
$$= (\nabla_{\theta'} \mathcal{L}_{\mathcal{T}_{i}}(f_{\theta'})) \cdot (\nabla_{\theta}(\theta - \alpha \nabla_{\theta} \mathcal{L}_{\mathcal{T}_{i}}(f_{\theta})))$$

- High computation cost
- Computation cost is increased with a number of inner-loop iterations T
- Use 1st order approximation

$$g_{\text{MAML}} = \nabla_{\theta} \mathcal{L}_{\mathcal{T}_{i}}(\theta') \approx (\nabla_{\theta'} \mathcal{L}_{\mathcal{T}_{i}}(f_{\theta'})) \cdot (\nabla_{\theta} \theta)$$
$$= \nabla_{\theta'} \mathcal{L}_{\mathcal{T}_{i}}(f_{\theta'})$$

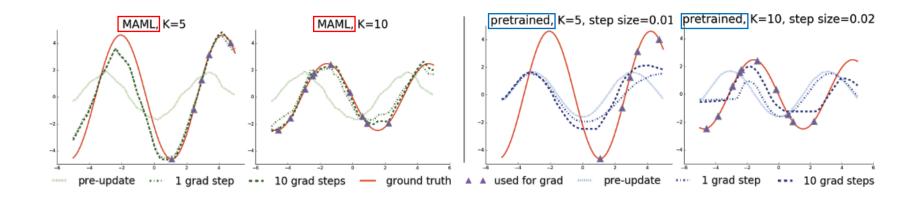
- Ignore 2nd order terms
- Empirically show similar performance

- Inner loop
 - One (or more) step of SGD on training loss starting from a meta-learned network
- Outer loop
 - Meta-parameters: initial weights of neural network
 - Meta-objective $\mathcal{L}_{\mathtt{mo}}$: validation loss
 - Meta-optimizer: SGD
- Learned model initial parameters adapt fast to new tasks

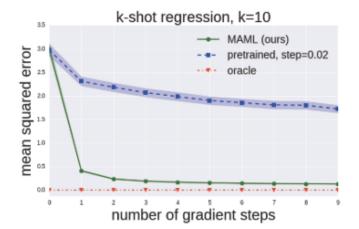


- Few-shot regression experiments
 - Regress the sine wave $y = A\sin(wx)$
 - Where $A \in [0.1, 5.0]$, $\ w \in [0, \pi]$, $\ x \in [-5, 5]$ are randomly sampled
 - MAML with one gradient update inner loop
 - Evaluate performance by fine-tuning the model
 - On *K*-samples, compared with simply pre-trained model

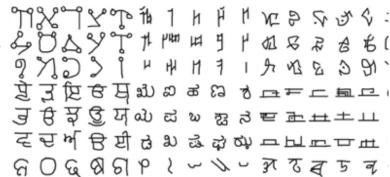
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 - Evaluate performance by fine-tuning the model
 - On K-samples, compared with simply pre-trained model
- Adapt much faster with small number of samples (purple triangle below)
 - MAML regresses well in the region without data (learn periodic nature of sine well)



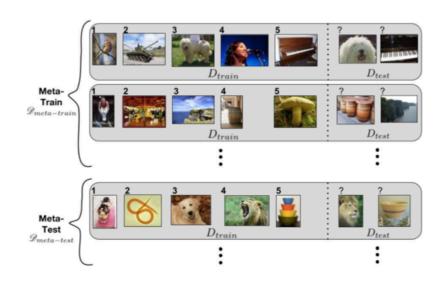
- Few-shot regression experiments
 - Regress the sine wave $y = A\sin(wx)$
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 - MAML with one gradient update inner loop
 - Evaluate performance by fine-tuning the model
 - On K-samples, compared with simply pre-trained model
- Adapt much faster with small number of samples (purple triangle below)
 - Continue to improve with additional gradient step
 - Not overfitted to θ that only improves after one step
 - Learn initialization that amenable to fast adaptation



- Datasets for few-shot classification task
- Omniglot
 - Various characters obtained from 50 alphabets
 - Consists of 20 samples of 1623 characters
 - 1200 meta-training, 423 meta-test classes



- Mini-Imagenet
 - Subset of ImageNet
 - 64 training, 12 validation, 24 test classes
 - For each class one/five samples are used



• Few-shot classification experiments

• Omniglot

	5-way Accuracy		20-way Accuracy	
Omniglot (Lake et al., 2011)	1-shot	5-shot	1-shot	5-shot
MANN, no conv (Santoro et al., 2016)	82.8%	94.9%	_	_
MAML, no conv (ours)	$89.7 \pm 1.1\%$	$97.5 \pm 0.6\%$	_	_
Siamese nets (Koch, 2015)	97.3%	98.4%	88.2%	97.0%
matching nets (Vinyals et al., 2016)	98.1%	98.9%	93.8%	98.5%
neural statistician (Edwards & Storkey, 2017)	98.1%	99.5%	93.2%	98.1%
memory mod. (Kaiser et al., 2017)	98.4%	99.6%	95.0%	98.6%
MAML (ours)	$98.7 \pm 0.4\%$	$99.9 \pm 0.1\%$	$95.8 \pm 0.3\%$	$98.9 \pm 0.2\%$

Mini-ImageNet

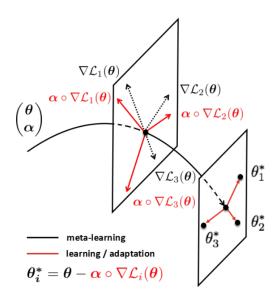
	5-way Accuracy	
MiniImagenet (Ravi & Larochelle, 2017)	1-shot	5-shot
fine-tuning baseline	$28.86 \pm 0.54\%$	$49.79 \pm 0.79\%$
nearest neighbor baseline	$41.08 \pm 0.70\%$	$51.04 \pm 0.65\%$
matching nets (Vinyals et al., 2016)	$43.56 \pm 0.84\%$	$55.31 \pm 0.73\%$
meta-learner LSTM (Ravi & Larochelle, 2017)	$43.44 \pm 0.77\%$	$60.60 \pm 0.71\%$
MAML, first order approx. (ours)	$48.07 \pm 1.75\%$	$63.15 \pm 0.91\%$
MAML (ours)	${\bf 48.70 \pm 1.84\%}$	$63.11 \pm 0.92\%$

MAML

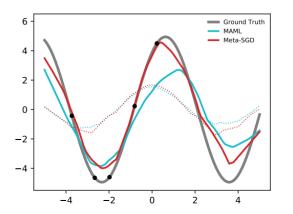
- MAML outperforms other baselines and generalizes well on unseen tasks
- It is model-agnostic
 - No dependency on network architectures
 - Can be used for another task not only few-shot learning (e.g., reinforcement learning)
 - Easily applicable to many applications
- Many recent works on meta-learning based on MAML
 - Learning the learning rate as well [Li, et. al., 2017]
 - First-order approximation of MAML [Nichol, et. al., 2018]
 - Probabilistic MAML [Finn, et. al., 2018]
 - Visual imitation learning [Finn, et. al., 2017]

An Extension: Meta-SGD - Learning Initialization and Learning Rates

- MAML uses the same learning rate for all the task
- Meta-SGD improves MAML by
 - Learning the learning rates for each task
 - Here the learning rates are vector, so that adjust the **gradient direction** as well
- Inner loop computation becomes: $\theta' = \theta \alpha \circ \nabla_{\theta} \mathcal{L}_{\mathcal{T}_i}(f_{\theta})$
 - Where α is a vector of learning rates



- Same few-shot regression experiment settings with MAML
 - By learning the hyperparameter (learning rates) Meta-SGD outperforms MAML



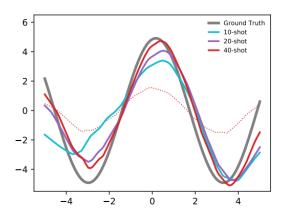


Figure 3: **Left:** Meta-SGD vs MAML on 5-shot regression. Both initialization (dotted) and result after one-step adaptation (solid) are shown. **Right:** Meta-SGD (10-shot meta-training) performs better with more training examples in meta-testing.

Table 1: Meta-SGD	vs MAML on	few-shot regression
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Meta-training	Models	5-shot testing	10-shot testing	20-shot testing
5-shot training	MAML	1.13 ± 0.18	0.85 ± 0.14	0.71 ± 0.12
J-shot training	Meta-SGD	$\boldsymbol{0.90 \pm 0.16}$	$\boldsymbol{0.63 \pm 0.12}$	$oldsymbol{0.50\pm0.10}$
10-shot training	MAML	1.17 ± 0.16	0.77 ± 0.11	0.56 ± 0.08
10-shot training	Meta-SGD	$\boldsymbol{0.88 \pm 0.14}$	$\boldsymbol{0.53 \pm 0.09}$	$oxed{0.35 \pm 0.06}$
20-shot training	MAML	1.29 ± 0.20	0.76 ± 0.12	0.48 ± 0.08
20-shot training	Meta-SGD	$\boldsymbol{1.01 \pm 0.17}$	$\boldsymbol{0.54 \pm 0.08}$	$\boldsymbol{0.31 \pm 0.05}$

Omniglot experiments

Table 2: Classification accuracies on Omniglot

	5-way Accuracy		20-way Accuracy	
	1-shot	5-shot	1-shot	5-shot
Siamese Nets	97.3%	98.4%	88.2%	97.0%
Matching Nets	98.1%	98.9%	93.8%	98.5%
MAML	$98.7 \pm 0.4\%$	$99.9 \pm 0.1\%$	$95.8 \pm 0.3\%$	$98.9 \pm 0.2\%$
Meta-SGD	$99.53 \pm 0.26\%$	$99.93 \pm 0.09\%$	$95.93 \pm 0.38\%$	$98.97 \pm 0.19\%$

Mini-Imagenet experiments

Table 3: Classification accuracies on MiniImagenet

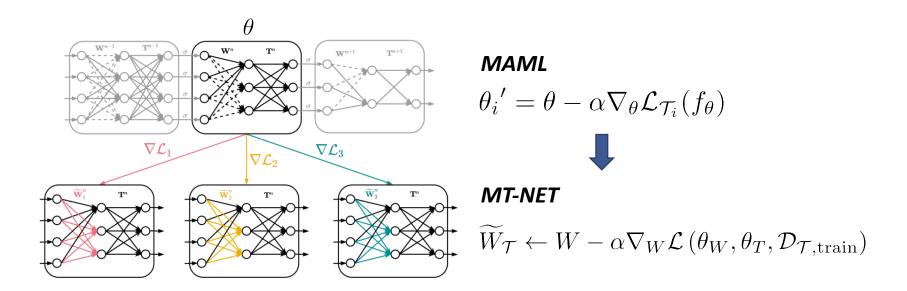
	5-way Accuracy		20-way Accuracy	
	1-shot	5-shot	1-shot	5-shot
Matching Nets	$43.56 \pm 0.84\%$	$55.31 \pm 0.73\%$	$17.31 \pm 0.22\%$	$22.69 \pm 0.20\%$
Meta-LSTM	$43.44 \pm 0.77\%$	$60.60 \pm 0.71\%$	$16.70 \pm 0.23\%$	$26.06 \pm 0.25\%$
MAML	$48.70 \pm 1.84\%$	$63.11 \pm 0.92\%$	$16.49 \pm 0.58\%$	$19.29 \pm 0.29\%$
Meta-SGD	$50.47 \pm 1.87\%$	${\bf 64.03 \pm 0.94\%}$	$f 17.56 \pm 0.64\%$	$28.92 \pm 0.35\%$

- Meta-SGD outperforms baselines with a large margin
 - Especially, it works well with many number of classes (20-way)

Meta-Learning for Learning Various Learning Rules

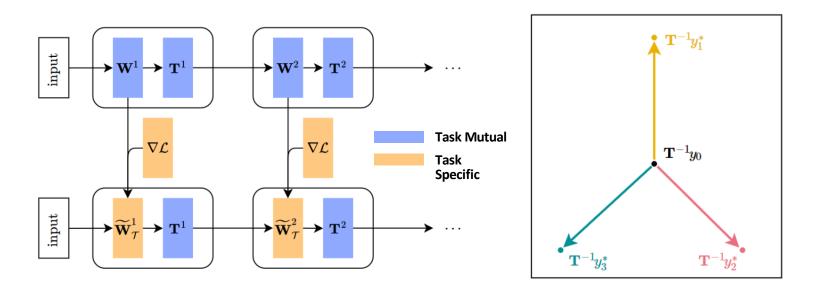
- Meta-SGD outperforms MAML in many experiments
 - Learning hyperparameter is useful as well
 - Indicate simple hyperparameter learning also gives benefit
- In many meta-learning methods meta-networks learn also:
 - Optimizer parameters: Learning rates, momentum, or optimizer itself
 - Metric space for data distribution similarity comparison
 - Weights of loss for each sample for handling data imbalance
 - And many other learning rules

 MT-NET [Choi et al. 18] proposes a MAML variant that chooses a subset of weights to fine-tune.



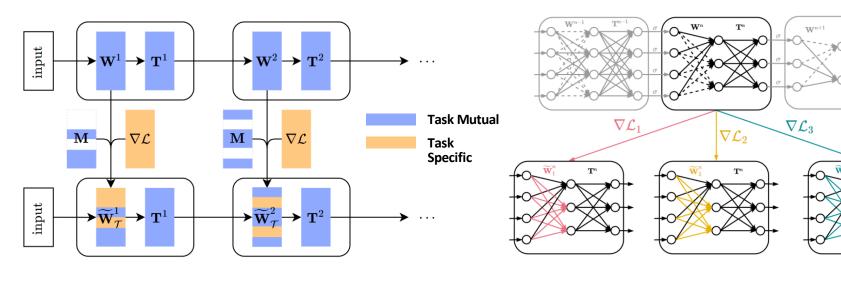
- A model f_{θ} consists of L cells, where each cell is parameterized as TW.
- The meta-learner specifies weights to be changed(dotted line) over initial weights(black) as chosen by task-specific learners(colored).

• A model f_{θ} consists of L cells, where each cell is parameterized as TW.

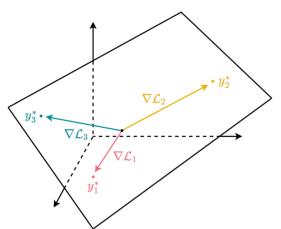


ullet T matrix learns a metric in activation space so that task specific weights W can preserve task identity.

 By adding binary mask, which selects weights to be updated, MT-NET chooses subspace that contributes to generalization.

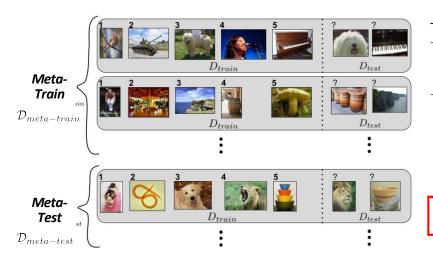


 Again, the meta-learner specifies subspace(dotted line) over initial weights(black) as chosen by taskspecific learners(colored).



Experiments-Classification

- Mini-ImageNet extracts 100 classes from ImageNet, and each class have 600 instances.
- MT-NET shows outperforming results over baselines.



Models	5-way 1-shot acc. (%)
Matching Networks(Vinyals et al., 2016) ¹	43.56 ± 0.84
Prototypical Networks (Snell et al., 2017) ²	46.61 ± 0.78
mAP-SSVM(Triantafillou et al., 2017)	50.32 ± 0.80
Fine-tune baseline ¹	28.86 ± 0.54
Nearest Neighbor baseline ¹	41.08 ± 0.70
meta-learner LSTM(Ravi & Larochelle, 2017)	43.44 ± 0.77
MAML(Finn et al., 2017)	48.70 ± 1.84
L-MAML(Grant et al., 2018)	49.40 ± 1.83
Meta-SGD(Li et al., 2017)	50.47 ± 1.87
T-net (ours)	50.86 ± 1.82
MT-net (ours)	$\textbf{51.70} \pm \textbf{1.84}$

Table of Contents

1. Introduction

- What is meta-learning?
- Applications of meta-learning
- Overview of common approaches

2. Approaches to Meta-learning

- Metric-based meta-learning
- Model-based meta-learning
- Optimization-based meta-learning
 - Learning model initialization
 - Learning optimizers

Optimizers for Learning DNNs

Learning DNNs is an optimization problem

$$\theta^* = \arg\min_{\theta} \mathcal{L}(\theta)$$

- \mathcal{L} be a task-specific objective (e.g., cross-entropy for classification)
- θ be parameters of a neural network

- How to find the optimal $heta^*$ which minimize $\mathcal L$?
 - The parameters are updated iteratively by taking gradient

$$\theta_{t+1} = \theta_t - \gamma \nabla \mathcal{L}(\theta_t)$$

- DNNs are often trained via "hand-designed" gradient-based optimizers
 - e.g., Nesterov momentum [Nesterov, 1983], Adagrad [Duchi et al., 2011], RMSProp [Tieleman and Hinton, 2012], ADAM [Kingma and Ba, 2015]

An Example of Optimizers: SGD with Momentum

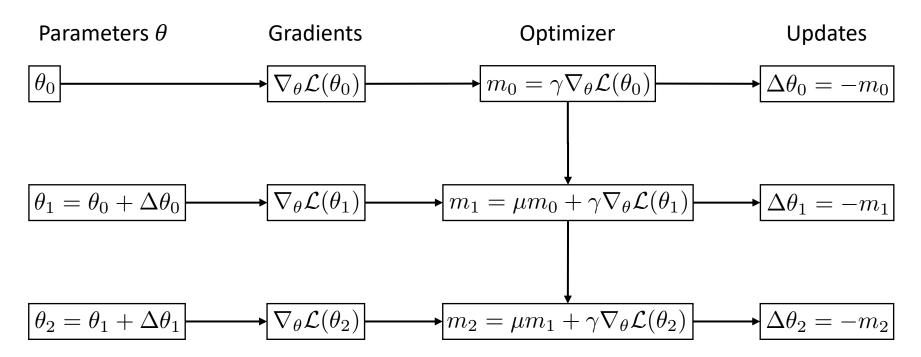
Update rules of SGD with momentum:

$$\theta_{t+1} = \theta_t - m_t$$

$$m_t = \mu m_{t-1} + \gamma \nabla_{\theta} \mathcal{L}(\theta_t)$$

where γ is a learning rate and μ is a momentum

Unroll the update steps



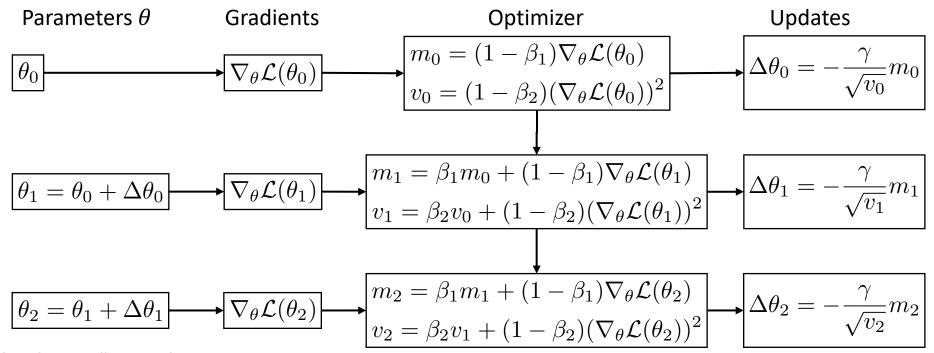
An Example of Optimizers: ADAM

Update rules of ADAM [Kingma and Ba, 2015]:

$$\theta_{t+1} = \theta_t - \frac{\gamma}{\sqrt{v_t}} m_t \qquad m_t = \beta_1 m_{t-1} + (1 - \beta_1) \nabla_{\theta} \mathcal{L}(\theta_t) \\ v_t = \beta_2 v_{t-1} + (1 - \beta_2) (\nabla_{\theta} \mathcal{L}(\theta_t))^2$$

where γ is a learning rate and β_1 , β_2 are decay rates for the moments

Unroll the update steps



No Free Lunch Theorem [Wolpert and Macready, 1997]

No algorithm is able to do better than a random strategy in expectation

- Drawbacks of these hand-designed optimizers (or update rules)
 - Potentially poor performance on some problems
 - Difficult to hand-craft the optimizer for every specific class of functions to optimize
- Solution: Learning an optimizer in an automatic way [Andrychowicz et al., 2016]
 - Explicitly model optimizers using recurrent neural networks (RNNs)

$$\theta_{t+1} = \theta_t + \underbrace{g_\phi(\nabla \mathcal{L}(\theta_t), h_t)}_{\text{Outputs of RNN}} \qquad h_t = f_\phi(\underbrace{\nabla \mathcal{L}(\theta_t)}_{\text{Inputs}}, \underbrace{h_{t-1}}_{\text{Hidden states}})$$

Cast an optimizer design as a learning problem

$$\phi^* = \arg\min_{\phi} \mathcal{L}(\theta_T(\phi))$$

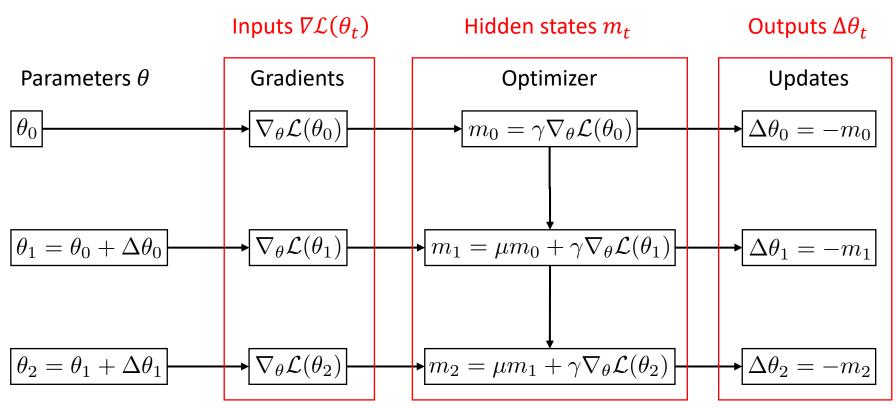
where $\theta_T(\phi)$ are the T-step updated parameters given the RNN optimizer ϕ

Update rules of SGD with momentum:

$$\theta_{t+1} = \theta_t - m_t$$

$$m_t = \mu m_{t-1} + \gamma \nabla_{\theta} \mathcal{L}(\theta_t)$$

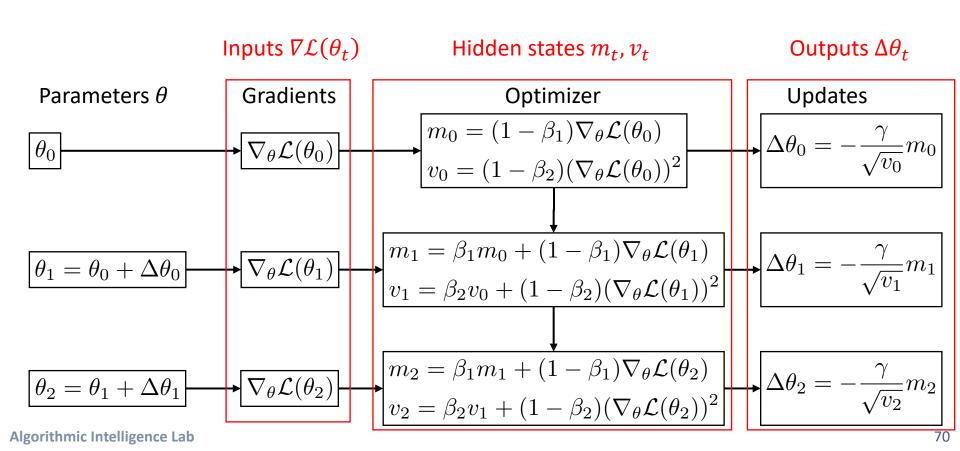
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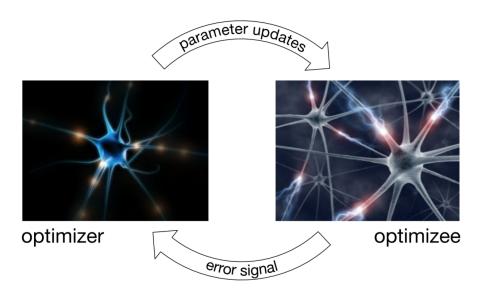
Update rules of ADAM [Kingma and Ba, 2015]:

$$\theta_{t+1} = \theta_t - \frac{\gamma}{\sqrt{v_t}} m_t \qquad m_t = \beta_1 m_{t-1} + (1 - \beta_1) \nabla_{\theta} \mathcal{L}(\theta_t) \\ v_t = \beta_2 v_{t-1} + (1 - \beta_2) (\nabla_{\theta} \mathcal{L}(\theta_t))^2$$

where γ is a learning rate and β_1 , β_2 are decay rates for the moments



[Andrychowicz et al. 16] proposes to learn the optimizer along with the learned model (optimizee).



$$\theta_{t+1} = \theta_t + g_t(\nabla f(\theta_t), \phi)$$

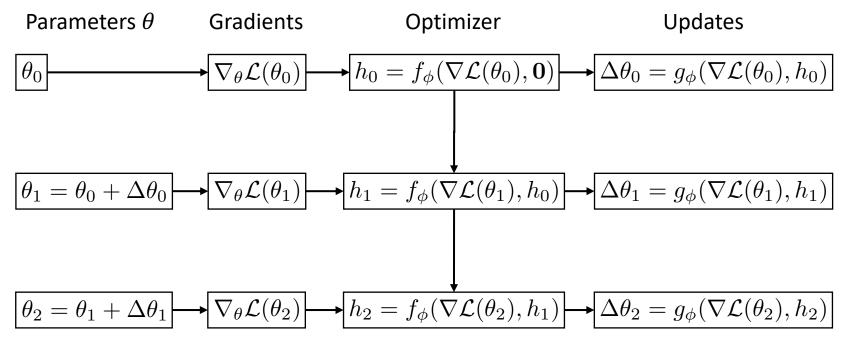
The optimizer could be thought as a neural network parameterized with ϕ that receives the gradient at step t as an input, and generates the update $\Delta\theta$.

RNN Optimizer

• Update rules based on a RNN $f_{m{\phi}}$, $g_{m{\phi}}$ parameterized by $m{\phi}$

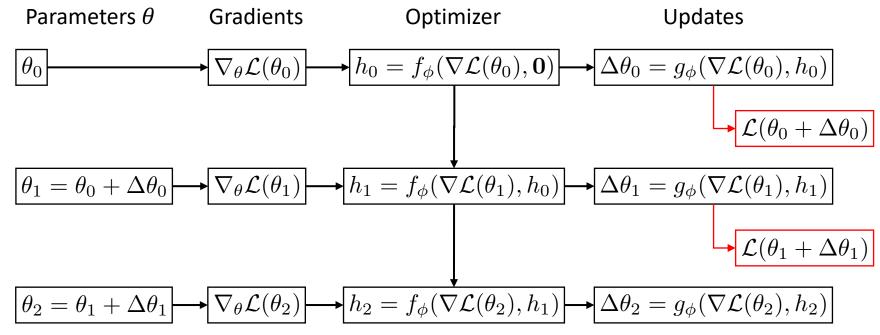
$$\theta_{t+1} = \theta_t + g_{\phi}(\nabla \mathcal{L}(\theta_t), h_t)$$
 $h_t = f_{\phi}(\nabla \mathcal{L}(\theta_t), h_{t-1})$

• Inner-loop: update the parameters θ via the optimizer for T times



• Objective for the RNN optimizer ϕ on the entire training trajectory

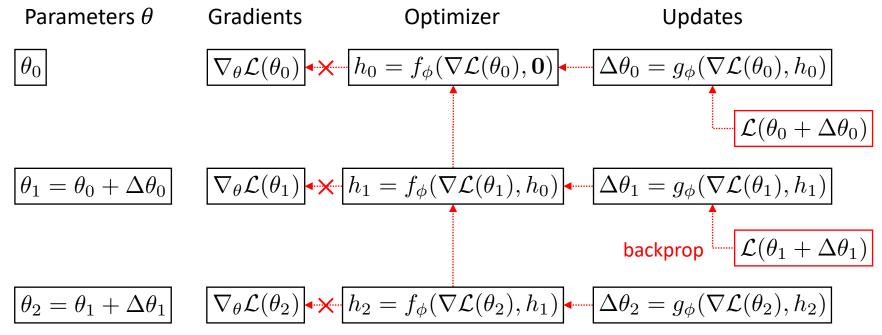
$$\mathcal{L}_{\mathtt{meta}}(\phi) = \sum_{t=1}^T w_t \mathcal{L}(heta_t)$$
 where w_t weights for each time-step



• Objective for the RNN optimizer ϕ on the entire training trajectory

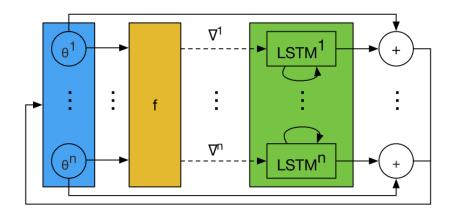
$$\mathcal{L}_{\mathtt{meta}}(\phi) = \sum_{t=1}^T w_t \mathcal{L}(\theta_t)$$
 where w_t weights for each time-step

- Outer-loop: minimize $\mathcal{L}_{ ext{meta}}(oldsymbol{\phi})$ using gradient descent on $oldsymbol{\phi}$
 - For simplicity, assume $\nabla_{\phi}\nabla_{\theta}\mathcal{L}(\theta_t)=0$ (then, only requires first-order gradients)



Architecture of RNN Optimizer

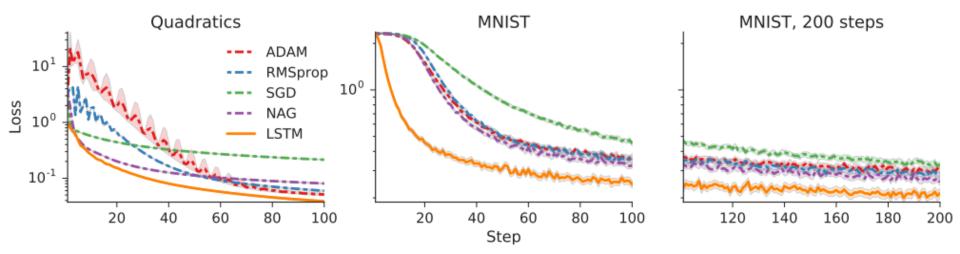
- A challenge is optimizing (at least) tens of thousands of parameters
 - Computationally not feasible with fully connected RNN architecture
- Use LSTM optimizer which operates coordinate-wise on the parameters
- By considering coordinate-wise optimizer
 - Able to use small network for optimizer
 - **Share optimizer parameters** across different parameters of the model
 - Input: gradient for single coordinate and the hidden state
 - Output: update for corresponding model parameter



- Learning models for
 - Quadratic functions

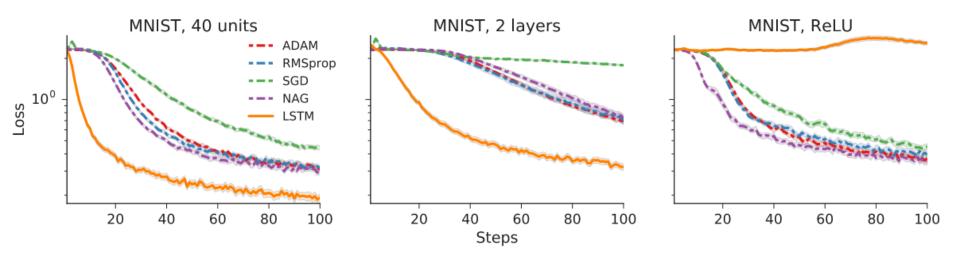
$$\mathcal{L}(\theta) = \|X\theta - y\|_2^2$$

- Optimizer is trained by optimizing random functions from this family
- Tested on newly sampled functions from the same distribution
- Neural network on MNIST dataset
 - Trained for 100 steps with MLP (1 hidden layer of 20 units, using a sigmoid function)
- Outperform baseline optimizers
 - Also perform well beyond the meta-trained steps (> 100 steps)

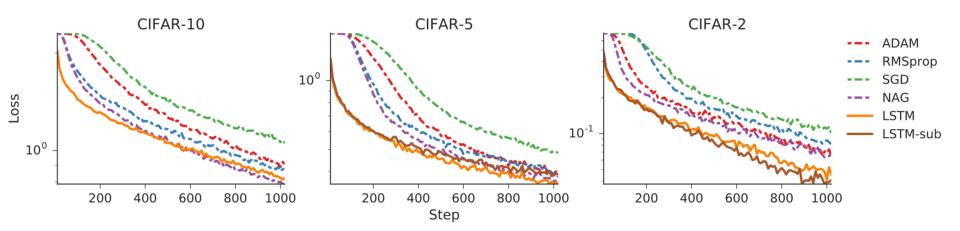


Generalization to different architecture models

- Learn LSTM optimizer for MNIST dataset
 - With 1 hidden layers (20 units) of sigmoid activation MLP
- Test generalization ability of a LSTM optimizer for
 - Different number of hidden units (20 \rightarrow 40)
 - Different number of hidden layers $(1 \rightarrow 2)$
 - Different **activation functions** (Sigmoid → ReLU)
- When learning dynamics are similar, the learned optimizer is generalized well
 - Different activation function significantly changes the problems to solve

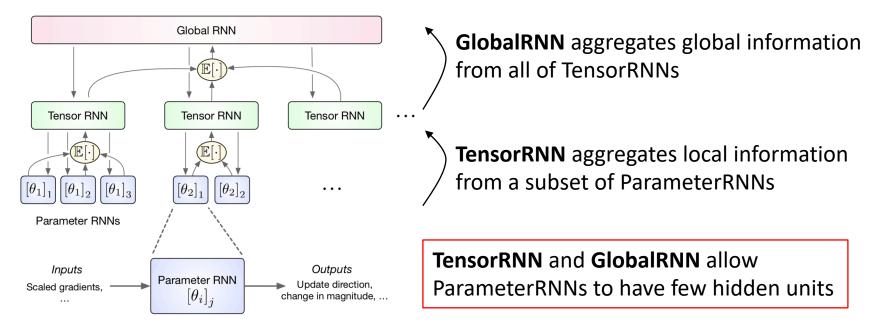


- **Generalization to different datasets**
 - Learn LSTM optimizer on CIFAR-10
 - Test on subset of CIFAR-10 (CIFAR-5 and CIFAR-2)
- Learn much faster than baseline optimizers
 - Even for different (but similar) dataset
 - Without additional tuning of the learned optimizer



An Extension: Hierarchical RNN Optimizer

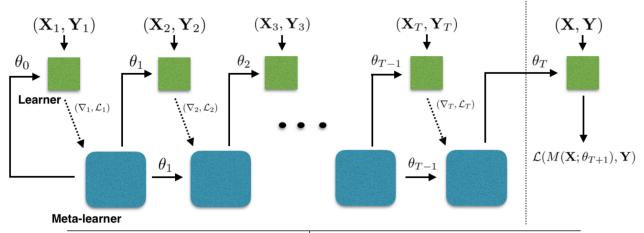
- Previous works have have difficulties in:
 - Large problems (e.g., large scale architecture, large number of steps)
 - Generalizing for various tasks
- To tackle these, hierarchical RNN is proposed [Wichrowska et al., 2017]



It generalizes to train Inception/ResNet on ImageNet for thousands of steps

Optimization as a Model for Few-shot Learning

- [Ravi and Larochelle17] used the learnable optimizer for few-shot learning.
- The meta-learning with learnable optimizer can be done by training it over multiple tasks.



Model	5-class		
Wiodei	1-shot	5-shot	
Baseline-finetune	$28.86 \pm 0.54\%$	$49.79 \pm 0.79\%$	
Baseline-nearest-neighbor	$41.08 \pm 0.70\%$	$51.04 \pm 0.65\%$	
Matching Network	$43.40 \pm 0.78\%$	$51.09 \pm 0.71\%$	
Matching Network FCE	$43.56 \pm 0.84\%$	$55.31 \pm 0.73\%$	
Meta-Learner LSTM (OURS)	$m{43.44 \pm 0.77\%}$	${\bf 60.60 \pm 0.71\%}$	

 The meta-learning optimizer (Meta-learner LSTM) outperforms Matching Networks for 5-shot cases.

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